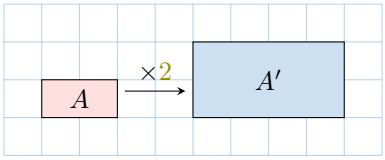
SIMILARITY

A DEFINITIONS

Discover: Examine the rectangles shown below. Although their sizes differ, they have the same shape because the proportions of their side lengths are identical.

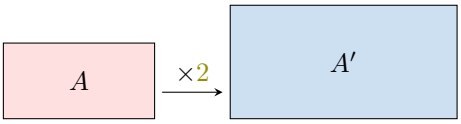


When A is enlarged to form A', the side lengths are doubled. The scale factor is 2.

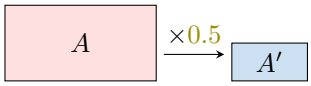
Definition **Similarity and Enlargement/Reduction**

A **similarity** is a transformation that multiplies all distances by a scale factor $k > 0$.

- If $k \geq 1$, the similarity is an **enlargement**.



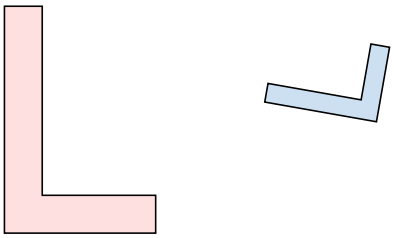
- If $0 < k < 1$, the similarity is a **reduction**.



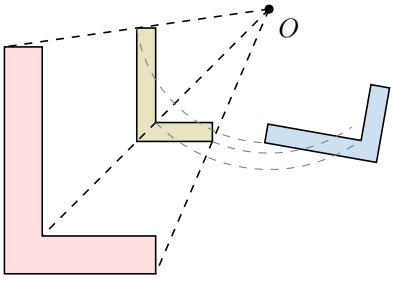
Theorem **Fundamental Transformations Similarity Theorem**

A similarity is the composition of one or more fundamental transformations (reflection, translation, rotation, and homothety).

Ex: The blue L is similar (by reduction) to the red L.



The blue L is the image of the red L through a homothety ($L \to L'$) followed by a rotation ($L' \to L$).

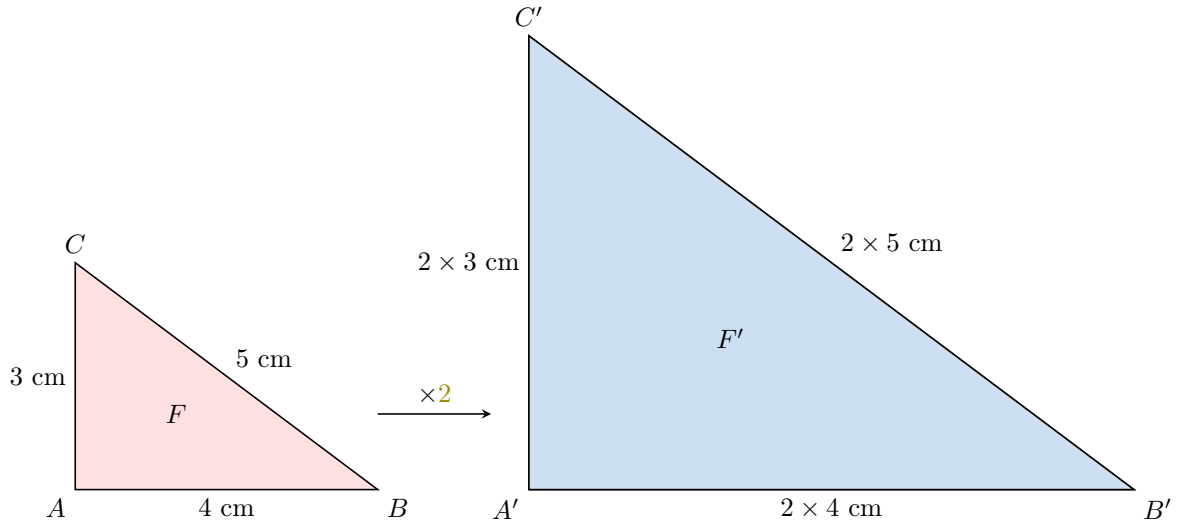


B SIMILAR FIGURES

Definition **Similar Figures**

Two figures are **similar** if one is an enlargement or reduction of the other.

Discover: The figure F' is an enlargement of the figure F by a scale factor of 2.



The ratios of the corresponding sides are:

- $\frac{A'B'}{AB} = \frac{2 \times 4 \text{ cm}}{4 \text{ cm}} = 2$
- $\frac{A'C'}{AC} = \frac{2 \times 3 \text{ cm}}{3 \text{ cm}} = 2$
- $\frac{B'C'}{BC} = \frac{2 \times 5 \text{ cm}}{5 \text{ cm}} = 2$

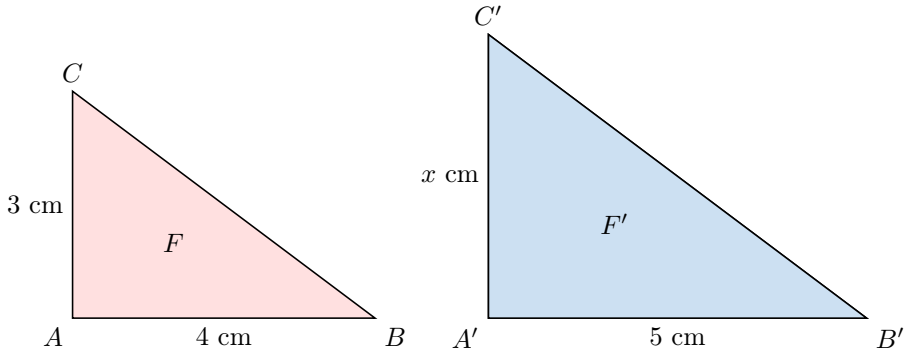
Thus, the ratios of the corresponding sides are equal to the scale factor.

Proposition Properties of Similar Figures

For similar figures:

- The ratios of the corresponding sides are equal to the scale factor.
- The corresponding angles are equal.

Ex: The figures F and F' are similar. Find x.



Answer: The ratios of the corresponding sides are equal:

$$\frac{A'C'}{AC} = \frac{A'B'}{AB}$$
$$\frac{x}{3} = \frac{5}{4}$$
$$x = 3 \times \frac{5}{4}$$
$$x = 3.75$$