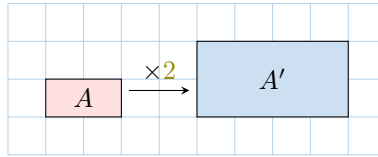# SIMILARITY

A WHAT IS A SIMILARITY?

Discover: Examine the rectangles shown below. Although their sizes differ, they have the same shape because the proportions of their side lengths are identical.
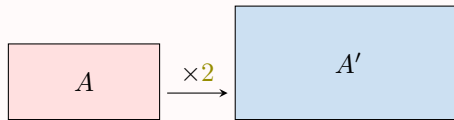

When A is enlarged to form A', the side lengths are doubled. The scale factor is 2.

Definition **Similarity and Enlargement/Reduction**

A **similarity** with scale factor $k > 0$ is a transformation that multiplies all distances by the same number k.

- If $k > 1$, the similarity is an **enlargement**.

- If $0 < k < 1$, the similarity is a **reduction**.

- If $k = 1$, the similarity preserves all distances (it is an isometry), so the figure has the same size and shape.
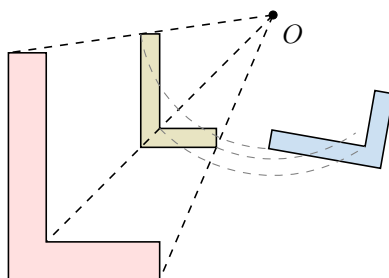

Theorem **Fundamental Transformations Similarity Theorem**

Any similarity can be expressed as the composition of one or more fundamental transformations (reflection, translation, rotation, and homothety).

Ex: The blue L is similar to the red L: it is a reduction of it.
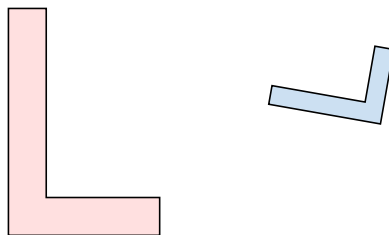
The blue L is the image of the red L through a homothety of scale factor 0.5 ($L \to L'$) followed by a rotation ($L' \to L$).
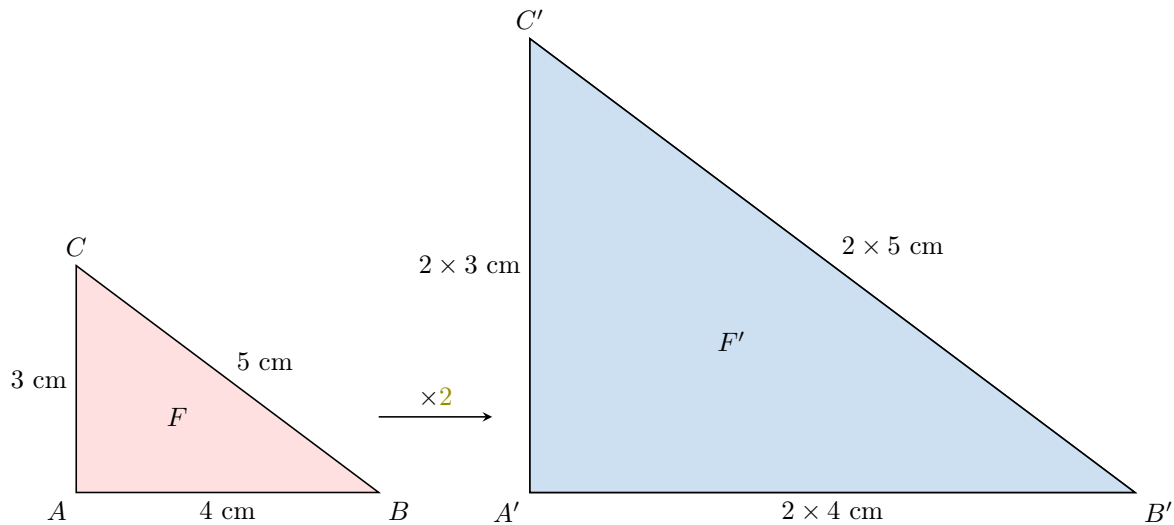
B SIMILAR FIGURES

> **Definition Similar Figures**
>
> Two figures are **similar** if one can be obtained from the other by a similarity (an enlargement, a reduction, or an isometry).

Discover: The figure F' is an enlargement of the figure F by a scale factor of 2.

The ratios of the corresponding sides are:

- $\dfrac{A'B'}{AB} = \dfrac{2 \times 4 \text{ cm}}{4 \text{ cm}} = 2$

- $\dfrac{A'C'}{AC} = \dfrac{2 \times 3 \text{ cm}}{3 \text{ cm}} = 2$

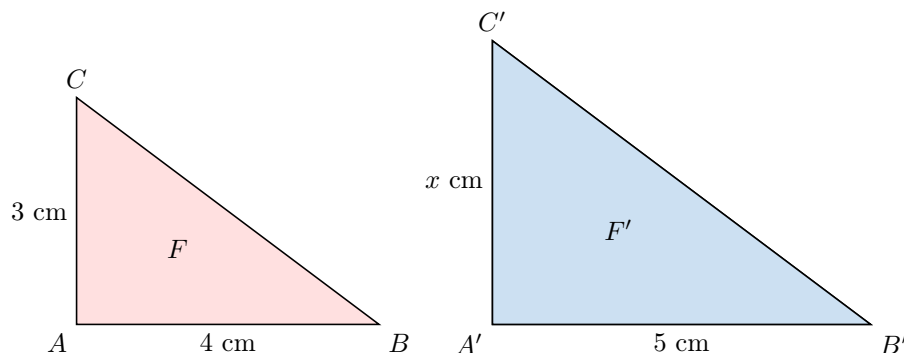- $\dfrac{B'C'}{BC} = \dfrac{2 \times 5 \text{ cm}}{5 \text{ cm}} = 2$

Thus, the ratios of the corresponding sides are equal to the scale factor.

> **Proposition Properties of Similar Figures**
>
> For similar figures:
>
> - The ratios of the lengths of corresponding sides are all equal to the same scale factor.
>
> - The corresponding angles are equal.

Ex: The figures F and F' are similar. Find x.

Answer: The ratios of the corresponding sides are equal:

$$\frac{A'C'}{AC} = \frac{A'B'}{AB}$$
$$\frac{x}{3} = \frac{5}{4}$$
$$x = 3 \times \frac{5}{4}$$
$$x = 3.75 \text{ cm}$$