

SOLVING EQUATIONS

A DEFINITIONS

Definition Solving an Equation

Solving an equation involves finding the values of the variable, called **solutions**, that make the equation true. In this context, the variable is called the **unknown**. We often use the letter x to represent the unknown.

Ex: Show that a solution of $3 + x = 5$ is $x = 2$.

Answer: For $x = 2$:

$$\begin{aligned} 3 + (2) &= 5 && \text{(substituting)} \\ 5 &= 5 && \text{(True)} \end{aligned}$$

Ex: Show that $x = 1$ is **not** a solution of $3 + x = 5$.

Answer: For $x = 1$:

$$\begin{aligned} 3 + (1) &= 5 && \text{(substituting)} \\ 4 &= 5 && \text{(False)} \end{aligned}$$

B SOLVING BY TRIAL AND ERROR

Method Trial and Error

The **trial and error method** is a problem-solving strategy used to find a solution to an equation by testing different values for the unknown variable until the correct value is found.

Ex: Consider the equation $2x + 3 = 11$.

Use the trial and error method to find a solution.

Answer:

- Let's try $x = 2$:

$$\begin{aligned} 2 \times (2) + 3 &= 11 && \text{(Substitute)} \\ 4 + 3 &= 11 \\ 7 &= 11 && \text{(False)} \end{aligned}$$

- Let's try $x = 3$:

$$\begin{aligned} 2 \times (3) + 3 &= 11 && \text{(Substitute)} \\ 6 + 3 &= 11 \\ 9 &= 11 && \text{(False)} \end{aligned}$$

- Let's try $x = 4$:

$$\begin{aligned} 2 \times (4) + 3 &= 11 && \text{(Substitute)} \\ 8 + 3 &= 11 \\ 11 &= 11 && \text{(True)} \end{aligned}$$

Therefore, a solution to the equation $2x + 3 = 11$ is $x = 4$.

C EQUIVALENT EQUATIONS

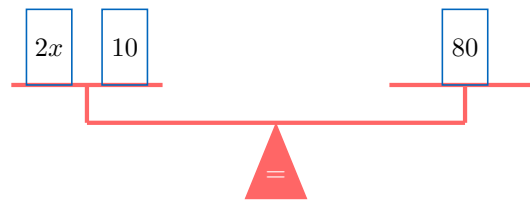
Definition Equivalent Equations

Two equations are **equivalent** if they have the same solutions.

An equation can be visually represented using the concept of a balance scale.

Definition Balance Scale

A **balance scale** has two pans, and when the weights on both pans are equal, the scale is in balance.



Representation of the equation $2x + 10 = 80$

Proposition Operations for Equivalent Equations

Equivalent equations are created by performing the same operation on both sides:

- Adding or subtracting the same number on both sides

Equation	Balance scale
$x + 10 = 80$	
Subtract 10 from both sides	Remove a weight of 10 from both pans
$x + 10 - 10 = 80 - 10$ $x = 70$	

- Multiplying or dividing both sides by the same (nonzero) number

Equation	Balance scale
$2x = 70$	
Divide both sides by 2	Divide the weight into two equal parts
$\frac{2x}{2} = \frac{70}{2}$ $x = 35$	

D DOING AND UNDOING EXPRESSIONS

Proposition Doing an Expression

To **do** an expression, perform the operations in order.

Ex: To **do** the expression $2x + 1$, start with x , multiply by 2, then add 1:

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+1} \boxed{2x + 1}$$

Definition Inverse Operations

- Adding and subtracting are inverse operations:

$$\boxed{x} \xrightarrow{+a} \boxed{x + a} \xrightarrow{-a} \boxed{x} \quad x + a = b \iff x = b - a$$

- Multiplying and dividing are inverse operations:

$$\boxed{x} \xrightarrow{\times a} \boxed{ax} \xrightarrow{\div a} \boxed{x} \quad a \times x = c \iff x = c \div a$$

Method Undoing an Expression

To **undo** an expression, use inverse operations in reverse order.

Ex: To **undo** the expression $2x + 1$, start with $2x + 1$, subtract 1, then divide by 2:

$$\boxed{2x + 1} \xrightarrow{-1} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$$

E SOLVING LINEAR EQUATIONS

Method Inverse Operations

If the unknown appears only once in the equation, we use **inverse operations** to simplify the expression and isolate the unknown.

Ex: Solve for x :

$$2x + 1 = 7$$

Answer: To solve $2x + 1 = 7$:

- To **do** the expression $2x + 1$:

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+1} \boxed{2x + 1}$$

- To **undo** the expression $2x + 1$:

$$\boxed{2x + 1} \xrightarrow{-1} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$$

- Step by step:

$$\begin{aligned} 2x + 1 &= 7 \\ 2x + 1 - 1 &= 7 - 1 && \text{(subtracting 1 from both sides)} \\ 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} && \text{(dividing both sides by 2)} \\ x &= 3 \end{aligned}$$

- So the solution is $x = 3$.