

SOLVING EQUATIONS

A DEFINITIONS

Discover: Welcome to the Math Escape Room! Today, you will embark on a journey to find the secret one-digit code. With your team, you must use your problem-solving skills to crack the code!
The code is:

$$\triangle + 10 = 1 + 2 \times 6$$

$$\triangle = \square$$

You have only one opportunity.

- Louis proposes to enter $\square 8$
- Su proposes to enter $\square 3$

Which code will you use to enter?

Answer: We will test each proposed code to determine which one satisfies the given equation.

- Test Louis's code: $\triangle = 8$

$$(8) + 10 = 1 + 2 \times 6 \quad (\text{Substitute})$$

$$18 = 1 + 12$$

$$18 = 13 \quad (\text{False})$$

- Test Su's code: $\triangle = 3$

$$(3) + 10 = 1 + 2 \times 6 \quad (\text{Substitute})$$

$$13 = 1 + 12$$

$$13 = 13 \quad (\text{True})$$

Therefore, the correct code to enter is: $\triangle = 3$.

Definition Solving an Equation

Solving an equation involves finding the values of the variable, called **solutions**, that make the equation true.

In this context, the variable is called the **unknown**.

We often use the letter x to represent the unknown.

Ex: Show that a solution of $3 + x = 5$ is $x = 2$.

Answer: For $x = 2$:

$$3 + (2) = 5 \quad (\text{substituting})$$

$$5 = 5 \quad (\text{True})$$

Ex: Show that $x = 1$ is **not** a solution of $3 + x = 5$.

Answer: For $x = 1$:

$$3 + (1) = 5 \quad (\text{substituting})$$

$$4 = 5 \quad (\text{False})$$

B SOLVING BY TRIAL AND ERROR

Method Trial and Error

The **trial and error method** is a problem-solving strategy used to find a solution to an equation by testing different values for the unknown variable until the correct value is found.

Ex: Consider the equation $2x + 3 = 11$.

Use the trial and error method to find a solution.

Answer:

- Let's try $x = 2$:

$$2 \times (2) + 3 = 11 \quad (\text{Substitute})$$

$$4 + 3 = 11$$

$$7 = 11 \quad (\text{False})$$

- Let's try $x = 3$:

$$\begin{aligned}
 2 \times (3) + 3 &= 11 && \text{(Substitute)} \\
 6 + 3 &= 11 \\
 9 &= 11 && \text{(False)}
 \end{aligned}$$

- Let's try $x = 4$:

$$\begin{aligned}
 2 \times (4) + 3 &= 11 && \text{(Substitute)} \\
 8 + 3 &= 11 \\
 11 &= 11 && \text{(True)}
 \end{aligned}$$

Therefore, a solution to the equation $2x + 3 = 11$ is $x = 4$.

C EQUIVALENT EQUATIONS

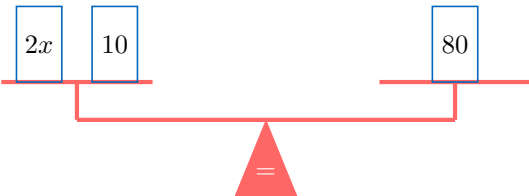
Definition Equivalent Equations

Two equations are **equivalent** if they have the same solutions.

An equation can be visually represented using the concept of a balance scale.

Definition Balance Scale

A **balance scale** has two pans, and when the weights on both pans are equal, the scale is in balance.



Representation of the equation $2x + 10 = 80$

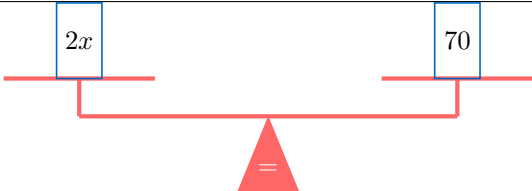
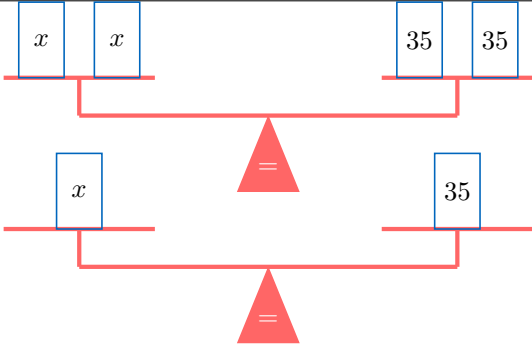
Proposition Operations for Equivalent Equations

Equivalent equations are created by performing the same operation on both sides:

- Adding or subtracting the same number on both sides

Equation	Balance scale
$x + 10 = 80$	
Subtract 10 from both sides	Remove a weight of 10 from both pans
$x + 10 - 10 = 80 - 10$ $x = 70$	

- Multiplying or dividing both sides by the same (nonzero) number

Equation	Balance scale
$2x = 70$	
Divide both sides by 2	Divide the weight into two equal parts
$\frac{2x}{2} = \frac{70}{2}$ $x = 35$	

D DOING AND UNDOING EXPRESSIONS

Proposition Doing an Expression

To **do** an expression, perform the operations in order.

Ex: To **do** the expression $2x + 1$, start with x , multiply by 2, then add 1:

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+1} \boxed{2x + 1}$$

Definition Inverse Operations

- Adding and subtracting are inverse operations:

$$\boxed{x} \xrightarrow{+a} \boxed{x+a} \xrightarrow{-a} \boxed{x} \quad x+a=b \iff x=b-a$$

- Multiplying and dividing are inverse operations:

$$\boxed{x} \xrightarrow{\times a} \boxed{ax} \xrightarrow{\div a} \boxed{x} \quad ax=c \iff x=c \div a$$

Method Undoing an Expression

To **undo** an expression, use inverse operations in reverse order.

Ex: To **undo** the expression $2x + 1$, start with $2x + 1$, subtract 1, then divide by 2:

$$\boxed{2x+1} \xrightarrow{-1} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$$

E SOLVING LINEAR EQUATIONS

Method Inverse Operations

If the unknown appears only once in the equation, we use **inverse operations** to simplify the expression and isolate the unknown.

Ex: Solve for x :

$$2x + 1 = 7$$

Answer: To solve $2x + 1 = 7$:

- To **do** the expression $2x + 1$:

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+1} \boxed{2x + 1}$$

- To **undo** the expression $2x + 1$:

$$\boxed{2x + 1} \xrightarrow{-1} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$$

- Step by step:

$$\begin{aligned} 2x + 1 &= 7 \\ 2x + 1 - 1 &= 7 - 1 && \text{(subtracting 1 from both sides)} \\ 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} && \text{(dividing both sides by 2)} \\ x &= 3 \end{aligned}$$

- So the solution is $x = 3$.

F SOLVING PRODUCT OF LINEAR EQUATIONS

Proposition Null Factor Law

When the product of two numbers is zero, at least one of them must be zero.

If $a \cdot b = 0$ then $a = 0$ or $b = 0$.

Ex: Solve for x : $x(x + 1) = 0$

Answer:

$$\begin{aligned} x(x + 1) &= 0 \\ x = 0 \text{ or } (x + 1) &= 0 \text{ (null factor law)} \\ x = 0 \text{ or } x &= -1 \end{aligned}$$

G SOLVING QUADRATIC EQUATIONS IN THE FORM $x^2 = k$

Discover: Consider the equation:

$$\begin{aligned} x^2 &= 9 \\ x^2 - 9 &= 0 \\ x^2 - 3^2 &= 0 \\ (x - 3)(x + 3) &= 0 && \text{(factoring by difference of squares)} \\ x - 3 = 0 \text{ or } x + 3 &= 0 && \text{(null factor law)} \\ x = 3 \text{ or } x &= -3 && \text{(solving linear equations)} \end{aligned}$$

Now consider:

$$x^2 = -9$$

The square x^2 is always positive or zero for real numbers, so it can never be equal to -9 . There is no real solution.

Proposition Solution of $x^2 = k$

If $x^2 = k$, then:

$$\begin{cases} x = \sqrt{k} \text{ or } x = -\sqrt{k} & \text{if } k > 0 \\ x = 0 & \text{if } k = 0 \\ \text{There are no real solutions} & \text{if } k < 0 \end{cases}$$

Ex: Solve for x : $x^2 = 3$

Answer: $x = \sqrt{3}$ or $x = -\sqrt{3}$