SOLVING EQUATIONS

A DEFINITIONS

Definition Solving an Equation

Solving an equation involves finding the values of the variable, called **solutions**, that make the equation true.
In this context, the variable is called the **unknown**.
We often use the letter x to represent the unknown.

Ex: Show that a solution of $3+x=5$ is $x=2$.

Answer: For $x=2$:

$$\begin{aligned} 3+(2) &= 5 && \text{(substituting)} \\ 5 &= 5 && \text{(True)} \end{aligned}$$

Ex: Show that $x=1$ is **not** a solution of $3+x=5$.

Answer: For $x=1$:

$$\begin{aligned} 3+(1) &= 5 && \text{(substituting)} \\ 4 &= 5 && \text{(False)} \end{aligned}$$

B SOLVING BY TRIAL AND ERROR

Method Trial and Error

The **trial and error method** is a problem-solving strategy used to find a solution to an equation by testing different values for the unknown variable until the correct value is found.

Ex: Consider the equation $2x+3=11$.
Use the trial and error method to find a solution.

Answer:

- Let's try $x=2$:

$$\begin{aligned} 2\times(2)+3 &= 11 && \text{(Substitute)} \\ 4+3 &= 11 \\ 7 &= 11 && \text{(False)} \end{aligned}$$

- Let's try $x=3$:

$$\begin{aligned} 2\times(3)+3 &= 11 && \text{(Substitute)} \\ 6+3 &= 11 \\ 9 &= 11 && \text{(False)} \end{aligned}$$

- Let's try $x=4$:

$$\begin{aligned} 2\times(4)+3 &= 11 && \text{(Substitute)} \\ 8+3 &= 11 \\ 11 &= 11 && \text{(True)} \end{aligned}$$

Therefore, a solution to the equation $2x+3=11$ is $x=4$.

C EQUIVALENT EQUATIONS

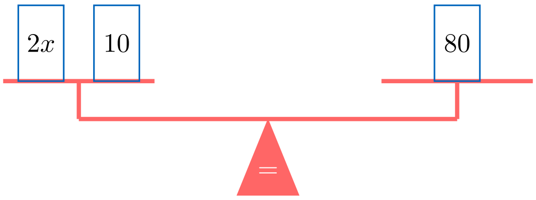
Definition Equivalent Equations

Two equations are **equivalent** if they have the same solutions.

An equation can be visually represented using the concept of a balance scale.

Definition Balance Scale

A **balance scale** has two pans, and when the weights on both pans are equal, the scale is in balance.

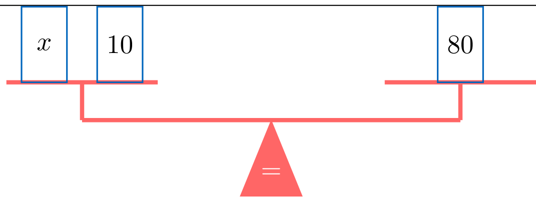
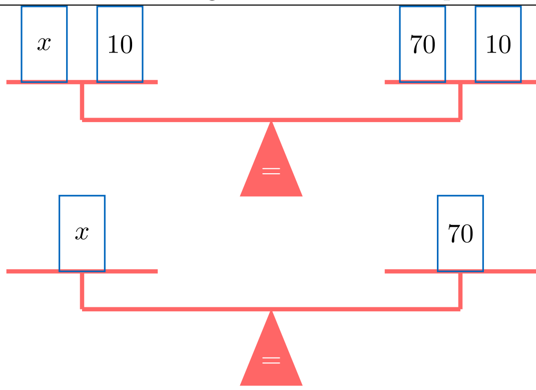


Representation of the equation $2x + 10 = 80$

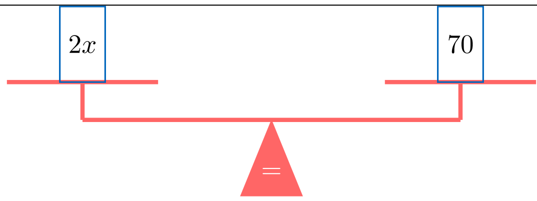
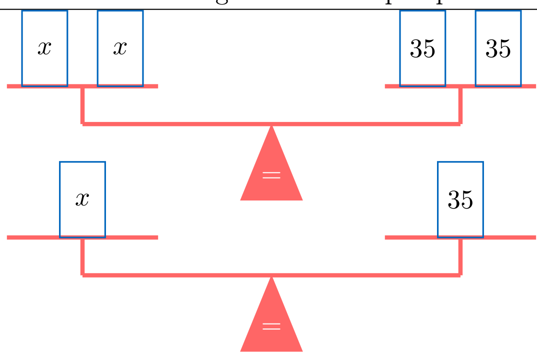
Proposition Operations for Equivalent Equations

Equivalent equations are created by performing the same operation on both sides:

- Adding or subtracting the same number on both sides

Equation	Balance scale
$x + 10 = 80$	
Subtract 10 from both sides	Remove a weight of 10 from both pans
$x + 10 - 10 = 80 - 10$ $x = 70$	

- Multiplying or dividing both sides by the same (nonzero) number

Equation	Balance scale
$2x = 70$	
Divide both sides by 2	Divide the weight into two equal parts
$\frac{2x}{2} = \frac{70}{2}$ $x = 35$	

D DOING AND UNDOING EXPRESSIONS

Proposition Doing an Expression

To **do** an expression, perform the operations in order.

Ex: To **do** the expression $2x+1$, start with x, multiply by 2, then add 1:

$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+1} \boxed{2x+1}$$

Definition Inverse Operations

- Adding and subtracting are inverse operations:
$\boxed{x} \xrightarrow{+a} \boxed{x+a} \xrightarrow{-a} \boxed{x} x+a=b \quad \Longleftrightarrow \quad x=b-a$
- Multiplying and dividing are inverse operations:
$\boxed{x} \xrightarrow{\times a} \boxed{ax} \xrightarrow{\div a} \boxed{x} a\times x=c \quad \Longleftrightarrow \quad x=c\div a$

Method Undoing an Expression

To **undo** an expression, use inverse operations in reverse order.

Ex: To **undo** the expression $2x+1$, start with $2x+1$, subtract 1, then divide by 2:

$$\boxed{2x+1} \xrightarrow{-1} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$$

E SOLVING LINEAR EQUATIONS

Method Inverse Operations

If the unknown appears only once in the equation, we use **inverse operations** to simplify the expression and isolate the unknown.

Ex: Solve for x:

$$2x+1=7$$

Answer: To solve $2x+1=7$:

- To **do** the expression $2x+1$:
$$\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+1} \boxed{2x+1}$$
- To **undo** the expression $2x+1$:
$$\boxed{2x+1} \xrightarrow{-1} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$$
- Step by step:
$$\begin{aligned} 2x+1&=7 \\ 2x+1-1&=7-1 && \text{(subtracting 1 from both sides)} \\ 2x&=6 \\ \frac{2x}{2}&=\frac{6}{2} && \text{(dividing both sides by 2)} \\ x&=3 \end{aligned}$$
- So the solution is $x=3$.

F SOLVING PRODUCT OF LINEAR EQUATIONS

Proposition Null Factor Law

When the product of two numbers is zero, at least one of them must be zero.

If $a\,b=0$ then $a=0$ **or** $b=0$.

Ex: Solve for x: $x(x+1)=0$

Answer:

$$x(x+1) = 0$$
$$x = 0 \text{ or } (x+1) = 0 \text{ (null factor law)}$$
$$x = 0 \text{ or } x = -1$$

G SOLVING QUADRATIC EQUATIONS IN THE FORM $x^2 = k$

Proposition Solution of $x^2 = k$

If $x^2 = k$, then:

$$\begin{cases} x = \sqrt{k} \text{ or } x = -\sqrt{k} & \text{if } k > 0 \\ x = 0 & \text{if } k = 0 \\ \text{There are no real solutions} & \text{if } k < 0 \end{cases}$$

Ex: Solve for x: $x^2 = 3$

Answer: $x = \sqrt{3}$ or $x = -\sqrt{3}$