SOLVING QUADRATIC EQUATIONS

A DEFINITION

A.1 IDENTIFYING COEFFICIENTS OF QUADRATIC EQUATIONS: LEVEL 1

Ex 1: For the equation $5x^2 - 2x - 3 = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$a =$ ☐, $b =$ ☐ and $c =$ ☐

Ex 2: For the equation $x^2 + 2x + 1 = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$a =$ ☐, $b =$ ☐ and $c =$ ☐

Ex 3: For the equation $-x^2 + 2 = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$a =$ ☐, $b =$ ☐ and $c =$ ☐

Ex 4: For the equation $-x^2 + 2x = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$a =$ ☐, $b =$ ☐ and $c =$ ☐

A.2 IDENTIFYING COEFFICIENTS OF QUADRATIC EQUATIONS: LEVEL 2

Ex 5: For the equation $x^2 - x + 3 = 1$, find the coefficients in the form $ax^2 + bx + c = 0$:

$a =$ ☐, $b =$ ☐ and $c =$ ☐

Ex 6: For the equation $(x + 1)^2 = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$a =$ ☐, $b =$ ☐ and $c =$ ☐

Ex 7: For the equation $(x - 2)^2 + 2 = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$a =$ ☐, $b =$ ☐ and $c =$ ☐

Ex 8: For the equation $x(x - 2) = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$a =$ ☐, $b =$ ☐ and $c =$ ☐

Ex 9: For the equation $(x - 2)(x + 1) = 0$, find the coefficients in the form $ax^2 + bx + c = 0$:

$a =$ ☐, $b =$ ☐ and $c =$ ☐

A.3 RECOGNIZING QUADRATIC EQUATIONS

MCQ 10: Is the equation $2x^2 - 3x + 2 = 0$ a quadratic equation?

☐ Yes.

☐ No.

MCQ 11: Is the equation $2x - 3 = 0$ a quadratic equation?

☐ Yes.

☐ No.

MCQ 12: Is the equation $2x^2 - 3x + \frac{1}{x} = 0$ a quadratic equation?

☐ Yes.

☐ No.

MCQ 13: Is the equation $(x - 1)(x + 2) = 0$ a quadratic equation?

☐ Yes.

☐ No.

A.4 VERIFYING ROOTS OF QUADRATIC EQUATIONS

MCQ 14: Is 1 a root of the equation $x^2 - 2x + 1 = 0$?

☐ Yes.

☐ No.

MCQ 15: Is 1 a root of the equation $x^2 + 2x + 1 = 0$?

☐ Yes.

☐ No.

MCQ 16: Is 2 a root of the equation $(x - 1)(x - 2) = 0$?

☐ Yes.

☐ No.

MCQ 17: Is 5 a root of the equation $(x - 2)^2 - 8 = 0$?

☐ Yes.

☐ No.

B SOLVING BY FACTORIZATION

B.1 FINDING SOLUTION SETS OF FACTORED QUADRATIC EQUATIONS

MCQ 18: For the equation $(x - 1)(x + 2) = 0$, the set of solutions is

☐ $S = \{-2, 1\}$

☐ $S = \{-1, 2\}$

☐ $S = \{2\}$

☐ $S = \{1\}$

MCQ 19: For the equation $x(x-\sqrt{2}) = 0$, the set of solutions is

☐ $S = \{0, -\sqrt{2}\}$

☐ $S = \{-\sqrt{2}\}$

☐ $S = \{0\}$

☐ $S = \{0, \sqrt{2}\}$

MCQ 20: For the equation $(x-1)^2 = 0$, the set of solutions is

☐ $S = \{1\}$

☐ $S = \{-1\}$

☐ $S = \{1, -1\}$

☐ $S = \{0\}$

MCQ 21: For the equation $(2x-1)(x+1) = 0$, the set of solutions is

☐ $S = \{-1, 2\}$

☐ $S = \left\{\frac{1}{2}, -1\right\}$

☐ $S = \{2\}$

☐ $S = \{-1\}$

B.2 SOLVING FACTORED QUADRATIC EQUATIONS

Ex 22: Solve the equation $(x-1)(x+2) = 0$. Justify your answer.

Ex 23: Solve the equation $(x+1)(x-1) = 0$. Justify your answer.

Ex 24: Solve the equation $((x-2)+3)((x-2)-3) = 0$. Justify your answer.

Ex 25: Solve the equation $(x+\sqrt{2})(x-\sqrt{2}) = 0$. Justify your answer.

C FACTORIZATION TECHNIQUES FOR SPECIAL FORMS OF EQUATIONS

C.1 FINDING SOLUTION SETS OF QUADRATIC EQUATIONS IN THE FORM $ax^2 + bx$

MCQ 26: For the equation $x^2 + x = 0$, the set of solutions is

☐ $S = \{-1, 0, 1\}$

☐ $S = \{1\}$

☐ $S = \{0\}$

☐ $S = \{0, -1\}$

MCQ 27: For the equation $x^2 - 2x = 0$, the set of solutions is

☐ $S = \{-2, 0\}$

☐ $S = \{2\}$

☐ $S = \{0\}$

☐ $S = \{0, 2\}$

MCQ 28: For the equation $2x^2 + x = 0$, the set of solutions is

☐ $S = \left\{-\frac{1}{2}, 0\right\}$

☐ $S = \{-2, 0\}$

☐ $S = \{2, 0\}$

☐ $S = \{0, 1\}$

MCQ 29: For the equation $3x^2 = x$, the set of solutions is

☐ $S = \{-3,\ 0\}$

☐ $S = \left\{0,\ \frac{1}{3}\right\}$

☐ $S = \{0,\ 3\}$

☐ $S = \{0,\ 1\}$

C.2 SOLVING QUADRATIC EQUATIONS IN THE FORM $ax^2 + bx$

Ex 30: Solve the equation $x^2 + x = 0$. Justify your answer.

Ex 31: Solve the equation $x^2 - 2x = 0$. Justify your answer.

Ex 32: Solve the equation $2x^2 - x = 0$. Justify your answer.

Ex 33: Solve the equation $2x^2 = 4x$. Justify your answer.

C.3 FINDING SOLUTION SETS OF QUADRATIC EQUATIONS IN THE FORM OF A DIFFERENCE OF SQUARES

MCQ 34: For the equation $x^2 - 4 = 0$, the set of solutions is

- ☐ $S = \{-4, 4\}$
- ☐ $S = \{2\}$
- ☐ $S = \{-1, 1\}$
- ☐ $S = \{-2, 2\}$

MCQ 35: For the equation $x^2 = -2$, the set of solutions is

- ☐ $S = \{-2, 2\}$
- ☐ $S = \{2\}$
- ☐ $S = \{\}$
- ☐ $S = \{-\sqrt{2}, \sqrt{2}\}$

MCQ 36: For the equation $x^2 - 2 = 0$, the set of solutions is

- ☐ $S = \{-\sqrt{2},\ \sqrt{2}\}$
- ☐ $S = \{-2, 2\}$
- ☐ $S = \{2\}$
- ☐ $S = \{-1, 1\}$

MCQ 37: For the equation $(x-1)^2 - 9 = 0$, the set of solutions is

- ☐ $S = \{-2, 4\}$
- ☐ $S = \{-3, 3\}$
- ☐ $S = \{2, 4\}$
- ☐ $S = \{-1, 1\}$

MCQ 38: For the equation $(x-1)^2 - 2 = 0$, the set of solutions is

- ☐ $S = \{1 - \sqrt{2},\ 1 + \sqrt{2}\}$
- ☐ $S = \{-2, 4\}$
- ☐ $S = \{-\sqrt{2}, \sqrt{2}\}$
- ☐ $S = \{-1, 1\}$

C.4 SOLVING QUADRATIC EQUATIONS IN THE FORM OF A DIFFERENCE OF SQUARES

Ex 39: Solve the equation $x^2 - 4 = 0$. Justify your answer.

Ex 40: Solve the equation $x^2 = -2$. Justify your answer.

Ex 41: Solve the equation $x^2 - 2 = 0$. Justify your answer.

Ex 42: Solve the equation $(x-1)^2 - 9 = 0$. Justify your answer.

Ex 43: Solve the equation $(x-1)^2 - 2 = 0$. Justify your answer.

D FACTORIZATION BY COMPLETING THE SQUARE

D.1 FINDING SOLUTION SETS OF QUADRATIC EQUATIONS

MCQ 44: For the equation $x^2 + 2x - 3 = 0$, the set of solutions is

- ☐ $S = \{-3, 1\}$
- ☐ $S = \{3, 1\}$
- ☐ $S = \{-1, 3\}$
- ☐ $S = \{-1, 1\}$

MCQ 45: For the equation $x^2 + 6x + 5 = 0$, the set of solutions is

- ☐ $S = \{-5, -1\}$
- ☐ $S = \{-5, 1\}$
- ☐ $S = \{-1, 5\}$
- ☐ $S = \{1, 5\}$

MCQ 46: For the equation $x^2 + 10x + 24 = 0$, the set of solutions is

- ☐ $S = \{2, 4\}$
- ☐ $S = \{2, 6\}$
- ☐ $S = \{4, 6\}$
- ☐ $S = \{-4, -6\}$

MCQ 47: For the equation $x^2 - 2x - 1 = 0$, the set of solutions is

- ☐ $S = \{1 - \sqrt{2}, 1 + \sqrt{2}\}$
- ☐ $S = \{-1, 2\}$
- ☐ $S = \{-1, 1\}$
- ☐ $S = \{0, 2\}$

D.2 SOLVING QUADRATIC EQUATIONS

Ex 48: Solve the equation $x^2 + 2x - 3 = 0$. Justify your answer.

Ex 49: Solve the equation $x^2 + 6x + 5 = 0$. Justify your answer.

Ex 50: Solve the equation $x^2 + 10x + 24 = 0$. Justify your answer.

Ex 51: Solve the equation $x^2 - 2x - 1 = 0$. Justify your answer.

E QUADRATIC FORMULA

E.1 CALCULATING THE DISCRIMINANT

Ex 52: For the equation $5x^2 - 2x - 3 = 0$, calculate the discriminant:

$\Delta =$ ☐

Ex 53: For the equation $x^2 + 6x + 5 = 0$, calculate the discriminant:

$\Delta =$ ☐

Ex 54: For the equation $2x^2 - x + 3 = 0$, calculate the discriminant:

$\Delta =$ ☐

Ex 55: For the equation $-2x^2 + 8 = 0$, calculate the discriminant:

$\Delta =$ ☐

E.2 SOLVING QUADRATIC EQUATIONS: STEP BY STEP

Ex 56: Consider the quadratic equation $x^2 + 2x - 3 = 0$.

1. Find the discriminant.

   $\Delta =$ ☐

2. Hence, state the nature of the roots of the equation.
   ☐ As $\Delta > 0$, there are 2 distinct roots.
   ☐ As $\Delta > 0$, there is 1 single root.
   ☐ As $\Delta > 0$, there are no roots.

3. The solutions of the equation are ☐ and ☐ (order from lowest to highest).

Ex 57: Consider the quadratic equation $x^2 - 2x - 1 = 0$.

1. Find the discriminant.

   $\Delta =$ ☐

2. Hence, state the nature of the roots of the equation.
   ☐ As $\Delta > 0$, there are 2 distinct roots.
   ☐ As $\Delta > 0$, there is 1 single root.
   ☐ As $\Delta > 0$, there are no roots.

3. The solutions of the equation are ☐ and ☐ (order from lowest to highest).

Ex 58: Consider the quadratic equation $2x^2 - 3x + 1 = 0$.

1. Find the discriminant.

   $\Delta =$ ☐

2. Hence, state the nature of the roots of the equation.
   ☐ As $\Delta > 0$, there are 2 distinct roots.
   ☐ As $\Delta > 0$, there is 1 single root.
   ☐ As $\Delta > 0$, there are no roots.

3. The solutions of the equation are ☐ and ☐ (order from lowest to highest).

Ex 59: Consider the quadratic equation $2x^2 - 4x + 2 = 0$.

1. Find the discriminant.

   $\Delta =$ ☐

2. Hence, state the nature of the roots of the equation.
   ☐ As $\Delta = 0$, there are 2 distinct roots.
   ☐ As $\Delta = 0$, there are no real roots.
   ☐ As $\Delta = 0$, there is 1 double root.

3. The solution of the equation is ☐.

E.3 SOLVING QUADRATIC EQUATIONS

Ex 60: Solve the quadratic equation $x^2 + 2x - 3 = 0$.

E.4 SOLVING QUADRATIC EQUATIONS

Ex 61: Solve the quadratic equation $x^2 + 2x - 2 = 0$.

Ex 62: Solve the quadratic equation $x^2 - 2x + 6 = 0$.

Ex 63: Solve the quadratic equation $x^2 - 6x + 9 = 0$.

Ex 64: Solve the quadratic equation $2x^2 - 5x + 2 = 0$.

Ex 65: Solve the quadratic equation $x^2 + 2x - 2 = 0$.

Ex 66: Solve the quadratic equation $x^2 - 8x + 15 = 0$.

Ex 67: Solve the quadratic equation $x^2 + 6x + 5 = 0$.