STATISTICS

A STATISTICAL INVESTIGATION

A.1 A STEP-BY-STEP GUIDE

Turn on the TV or flip through a newspaper, and you'll often spot statistics in action. For example:

- Su averages 14.6 points per basketball game.
- Last year was the hottest on record since 1897.

Statistics is the science of gathering, organizing, analyzing, interpreting, and presenting data. It helps us make smart decisions in all kinds of areas. Check out these real-world examples:

- **Scientific Research**: Testing if a new medicine works by studying trial results.
- **Industrial Production**: Improving products by tracking defects and fixing processes.
- **Social Issues**: Figuring out what people think about new laws through surveys.

A statistical investigation follows these five steps:

- **Step 1: State the Problem**: Decide what you want to learn.
  Example: How has the average temperature changed over the last 100 years?
- **Step 2: Collect Data**: Gather the info you need.
  Example: Get temperature records from weather stations.
- **Step 3: Calculate Descriptive Statistics**: Summarize the data with tools like mean, median, or mode.
  Example: Find the average temperature for each decade.
- **Step 4: Organize and Display Data**: Put the data in order and show it with charts or graphs.
  Example: Make a graph of temperature changes over time.
- **Step 5: Interpret the Statistics**: Figure out what the data tells you.
  Example: Does the data show temperatures are rising significantly?

By following these steps, you can dig into data and use it to make solid decisions!

Definition **Statistics**

Statistics is all about collecting information, sorting it out, summarizing it, and figuring out what it means.

B STATING THE PROBLEM

B.1 POPULATION

When you start a statistical investigation, the first step is to ask a clear question. This keeps you focused on what you're trying to find out and who or what you're studying.
We call the group we're studying the **population**. It could be all the people in a country, every student in a school, all the animals of a species, or even every item made by a machine. The information we collect from this group is called **data**, and it can come in many forms—like numbers, words, or measurements.

Definition **Problem**

A **problem** in statistics is a question that guides us to the information we need to find.

Ex: Do girls like math more than boys?

Definition **Population**

A **population** is the whole group of people or things with something in common that we want to study.

Ex: The population is all the students in a college.

B.2 DATA

Definition **Data**

Data is the information we collect, like numbers, words, measurements, or observations.

Ex: For our math study, we collect:

- **Gender**: Is the student a boy or a girl?
- **Favorite Subject**: What subject do they like best (e.g., Math, Science, English)?
- **Math test score**: What was their grade on the last assessment??

Definition **Types of Variables**

- **Qualitative Variable (Categorical)**: Describes categories or groups that cannot be measured numerically.
- **Quantitative Variable (Numerical)**: Represents measurable quantities with numerical values.

Ex: For our math study:

- **Qualitative Variables**: Gender and favorite subject.
- **Quantitative Variable**: Math test score.

C COLLECTING DATA

C.1 SAMPLING

To collect data, we first decide who or what we're asking. We can either:

- Do a **census**: Ask every single member of the population.
- Do a **survey**: Ask just a part of the population (a sample).

Why choose a survey? A census takes a lot of time and money, especially for big groups. A survey is faster and cheaper, and if we pick the sample well, it can still tell us a lot about the whole population!

Definition **Census**

A **census** means collecting data from everyone in the population.

Definition **Survey**

A **survey** means collecting data from a smaller group (sample) of the population.

Ex: If you ask every student in the collège about their favorite subject, is it a census or a survey?

Answer: It's a census.

Ex: If you only ask the students who are in class math today, is it a census or a survey?

Answer: It's a survey.

C.2 STATISTICAL ERROR IN SAMPLING

One of the most common ways to collect information about a large group is to use a sample.For a sample to be meaningful, it must fairly represent the entire population.Two key challenges in sampling are: avoiding bias and ensuring the sample is large enough to capture the population's diversity.

- **Selection bias**: A famous example of biased sampling is the Literary Digest poll before the 1936 U.S. presidential election.The magazine sent millions of surveys using telephone books and car registration lists. But during the Great Depression, many people couldn't afford phones or cars.This led to a sample biased toward wealthier citizens, who were more likely to vote Republican.As a result, the poll incorrectly predicted a landslide win for Alfred Landon, while Franklin D. Roosevelt won by a wide margin.
- **Sample size**: During the Cuban Missile Crisis of 1962, U.S. intelligence underestimated the number and types of Soviet missiles in Cuba due to limited reconnaissance data.The small "sample" of photos led analysts to miss several launch sites, including those with longer-range missiles.This example shows how insufficient data can lead to serious misjudgments, especially when the stakes are high.

Definition **Statistical Error**

A **statistical error** is the difference between the observed result (from the sample) and the actual value (in the population).

Definition **Selection Bias**

Selection bias occurs when the sampling method makes some individuals in the population less likely to be included than others.

Proposition **Random Sampling**

If each member of the population is selected randomly, selection bias is avoided.

Proposition **Sample Size**

As the sample size increases, the statistical error generally decreases—our results become more accurate.

D DESCRIPTIVE STATISTICS

D.1 A STATISTIC

Descriptive statistics are numbers that help us summarize and understand data—like finding the average or the most common answer.

Definition **A statistics**

A **statistics** is a single value that sums up or describes a set of data.

Ex: The average score in a class is 85% is a statistics number because it tells us something about the whole group in one simple figure.

D.2 RELATIVE FREQUENCY

In statistics, it's important to understand the frequency of a category. This concept helps us analyze patterns and make predictions. It applies to everyday scenarios, such as gauging the popularity of a favorite food among friends or calculating how often a basketball player scores a shot. By studying relative frequencies, we gain valuable insights into data trends.

Definition **Frequency and Relative Frequency**

Frequency is how many times each value or category appears.
Relative Frequency is the frequency divided by the total, often shown as a percentage.

Ex: The data for favorite subject is: Maths: 15 students, Sciences: 12 students, English: 3 students.
Fill in the table:

Subject	Frequency	Relative frequency (%)
Maths		
Sciences		
English		
Total		100%

Answer:

Subject	Frequency	Relative frequency (%)
Maths	15	$\frac{15}{30} \times 100\% = 50\%$
Sciences	12	$\frac{12}{30} \times 100\% = 40\%$
English	3	$\frac{3}{30} \times 100\% = 10\%$
Total	30	100%

D.3 CENTRAL TENDENCY

In statistics, central tendency refers to a measure that identifies a single value as representative of the center or typical point of a dataset. Three key measures are commonly used to assess central tendency: the mode, the mean, and the median.

Definition **Mode**

The **mode** is the value that shows up most often in your data.

Ex: A group of students reported their last mark (out of 5) on a math exam as follows:

$$1, 4, 2, 3, 5, 4, 5, 4, 3$$

What is the mode of this dataset?

Answer: From the frequency table:

Mark	Frequency
1	1
2	1
3	2
4	4
5	2

The mode is 4 because it appears most frequently (4 times).

Definition **Mean**

The **mean** is the average. Add up all the values and divide by how many there are:

$$\begin{aligned}\bar{x} &= \frac{\text{sum of all values}}{\text{number of values}} \\ &= \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}\end{aligned}$$

Ex: Ratings: 1, 4, 2, 3, 5, 4, 5, 4, 4. What's the mean?

Answer:

$$\begin{aligned}\text{Mean} &= \frac{1+4+2+3+5+4+5+4+4}{9} \\ &= \frac{32}{9} \\ &\approx 3.56\end{aligned}$$

Definition **Median**

The **median** is the middle value when you line up the data from smallest to largest:

- If there's an odd number of values, it's the one in the middle.
- If there's an even number, average the two middle values.

Ex: Ratings: 1, 4, 2, 3, 5, 4, 5, 4, 4. What's the median?

Answer: The ordered set is:

$$1, 2, 3, 4, 4, 4, 4, 5, 5$$

The middle value is 4, so the median is 4.

D.4 DISPERSION

When analyzing data, it's not only important to understand the **central tendency**—which refers to the typical value of a dataset (such as the mean, median, or mode)—but also to examine how much the data varies. This variation is called **dispersion**.

While measures of central tendency summarize the center of the data, measures of dispersion tell us how spread out the values are.To illustrate this, let's look at the test scores of two students:

- **Student A's scores:** 10, 50, 90
- **Student B's scores:** 45, 50, 55

Both students have the same mean score of 50. However, their scores are distributed differently:

- **Student A's scores:** show a wide variation, ranging from 10 to 90.
- **Student B's scores:** are much more concentrated, between 45 and 55.

This example shows that even when two datasets have the same average, their distributions can be very different. Measures of dispersion, such as the range and interquartile range, help us better understand this variability.

Definition Range

The **range** is the difference between the maximum and minimum values in a dataset.

$$\text{range} = \text{maximum} - \text{minimum}$$

Ex: Find the range for the following data: $1, 19, 10, 2, 18, 10, 5, 15, 10$.

Answer: The minimum value is 1 and the maximum is 19.
So, the range is $19 - 1 = 18$.

Definition Quartile

Quartiles are values that divide an ordered dataset into four equal parts.

The median splits the data into two halves. The quartiles divide these halves again, giving us four equal parts.

Definition Interquartile Range

The **interquartile range (IQR)** is the difference between the upper quartile (Q3) and the lower quartile (Q1).

$$\text{interquartile range} = \text{Q3} - \text{Q1}$$

Ex: Find the quartiles and the interquartile range for the following data:

$$1, 19, 10, 2, 18, 10, 5, 15, 10$$

Answer:

- Order the data:
$$1, 2, 5, 10, 10, 10, 15, 18, 19$$
- The median (Q2) is 10.
- The lower half (before the median): $1, 2, 5, 10 \rightarrow Q_1 = \frac{2+5}{2} = 3.5$
- The upper half (after the median): $10, 15, 18, 19 \rightarrow Q_3 = \frac{15+18}{2} = 16.5$
- So, the interquartile range is $16.5 - 3.5 = 13$

E ORGANIZING AND DISPLAYING DATA

E.1 VISUALIZING FREQUENCIES

Definition Bar Chart/Histogram

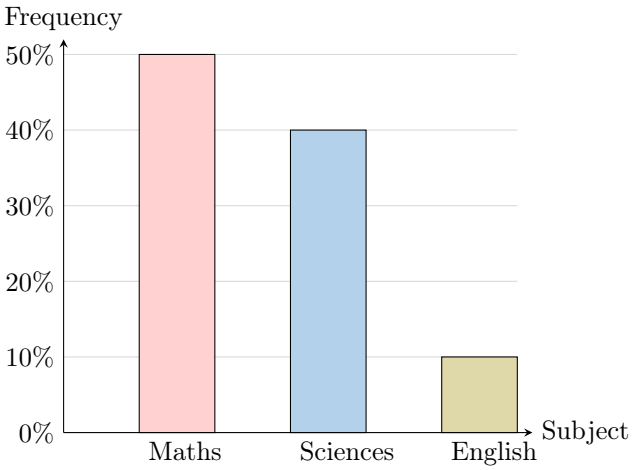
A **bar chart/histogram** shows data with bars:

- Categories or values go on x-axis.
- Frequencies go on y-axis.

Ex: Draw a bar chart for:

Subject	Relative frequency (%)
Maths	50%
Sciences	40%
English	10%

Answer:



Definition **Pie Chart**

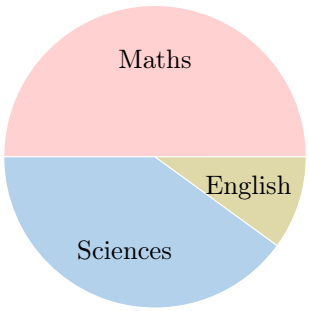
A **pie chart** is a circle split into slices to show how data compares.

Ex: Draw the pie chart of the following data:

Subject	Frequency
Maths	15
Sciences	12
English	3
Total	30

Answer: Angles are :

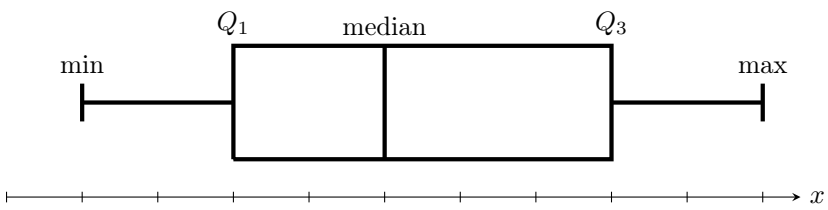
- Maths : $\frac{15}{30} \times 360° = 180°$
- Sciences : $\frac{12}{30} \times 360° = 144°$
- English : $\frac{3}{30} \times 360° = 36°$



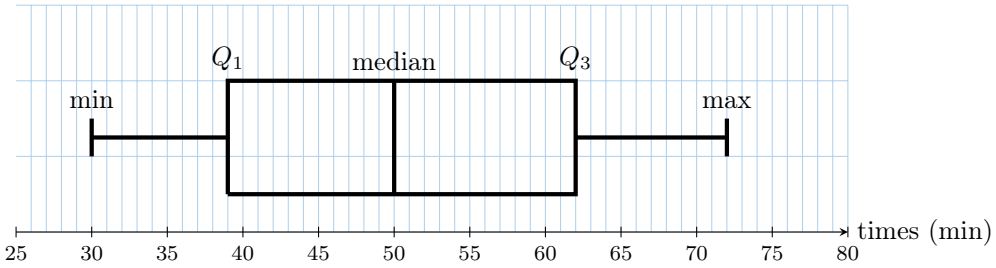
E.2 VISUALIZING CENTRAL TENDENCY AND DISPERSION

Definition **Box plot (whisker plot)**

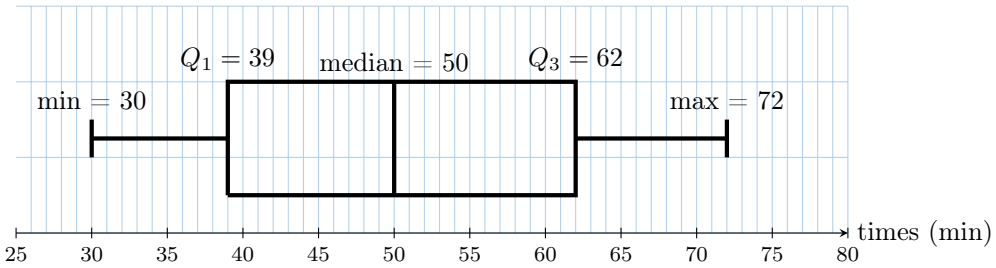
A **whisker plot**, also called a **box plot**, displays the five-number summary of a set of data. The five-number summary is the minimum, first quartile, median, third quartile, and maximum.
In a box plot, we draw a box from the first quartile to the third quartile. A vertical line goes through the box at the median. The whiskers go from each quartile to the minimum or maximum.



Ex: This box plot shows the number of minutes passengers spent in an airport departure lounge. What is the minimum number of minutes spent waiting in the lounge?



Answer:

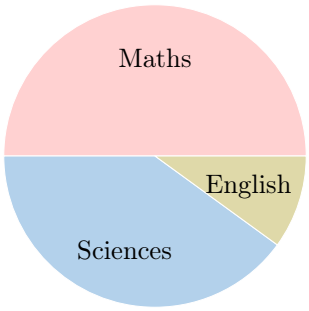


The minimum time is 30 minutes.

F INTERPRETING THE STATISTICS

F.1 READING AND COMPARING DATA

Ex: This pie chart shows the favorite subjects of students:



What does this chart suggest?

Answer: The largest section corresponds to **Maths**, so it is the most popular subject.

Ex: The girls' average score in math is 87 (B+), while the boys' average is 75 (C). Are girls better at math?

Answer: Yes, since $87 > 75$, on average, girls perform better than boys in math.