TRANSFORMATIONS

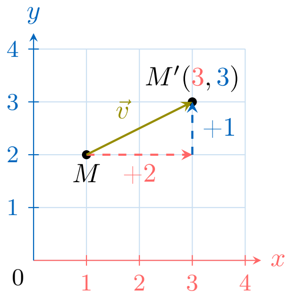
A TRANSLATION

A.1 DETERMINING THE IMAGE UNDER A TRANSLATION

Ex 1: Find the coordinates of the image of point $M(1,2)$ under a translation by vector $\vec{v}\begin{pmatrix}2\\1\end{pmatrix}$.

$$M'(\boxed{3},\boxed{3})$$

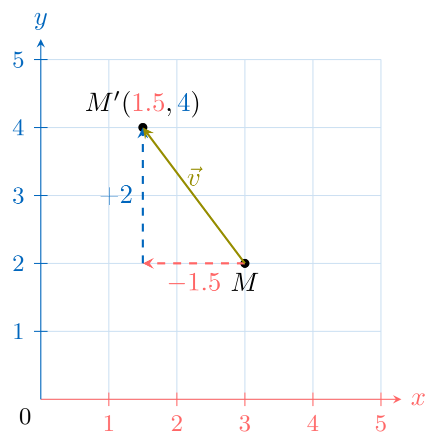
Answer: $M'(1+2, 2+1)$ so $M'(3,3)$



Ex 2: Find the coordinates of the image of point $M(3,2)$ under a translation by vector $\vec{v}\begin{pmatrix}-1.5\\2\end{pmatrix}$.

$$M'(\boxed{1.5},\boxed{4})$$

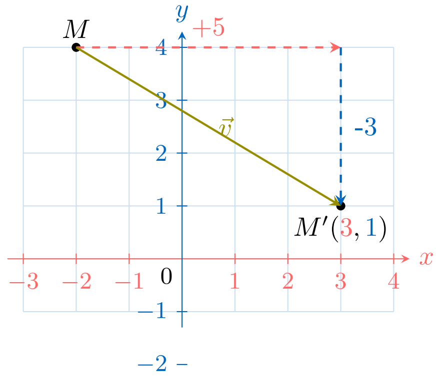
Answer: $M'(3-1.5, 2+2)$ so $M'(1.5,4)$



Ex 3: Find the coordinates of the image of point $M(-2,4)$ under a translation by vector $\vec{v}\begin{pmatrix}5\\-3\end{pmatrix}$.

$$M'(\boxed{3},\boxed{1})$$

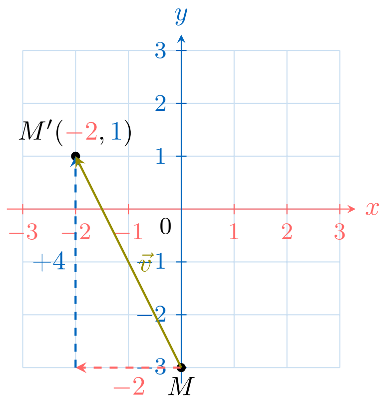
Answer: $M'(-2+5, 4-3)$ so $M'(3,1)$



Ex 4: Find the coordinates of the image of point $M(0,-3)$ under a translation by vector $\vec{v}\begin{pmatrix}-2\\4\end{pmatrix}$.

$$M'(\boxed{-2},\boxed{1})$$

Answer: $M'(0-2, -3+4)$ so $M'(-2,1)$



A.2 DETERMINING THE ORIGINAL POINT UNDER A TRANSLATION

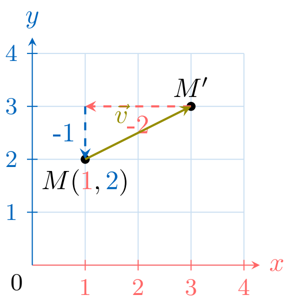
Ex 5: Find the coordinates of the point M whose image is $M'(3,3)$ under a translation by vector $\vec{v}\begin{pmatrix}2\\1\end{pmatrix}$.

$$M(\boxed{1},\boxed{2})$$

Answer:

$$\overrightarrow{MM'} = \vec{v}$$
$$\begin{pmatrix}3-x\\3-y\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$$
$$3-x=2 \text{ and } 3-y=1$$
$$x=3-2 \text{ and } y=3-1$$
$$x=1 \text{ and } y=2$$

so $M(1,2)$



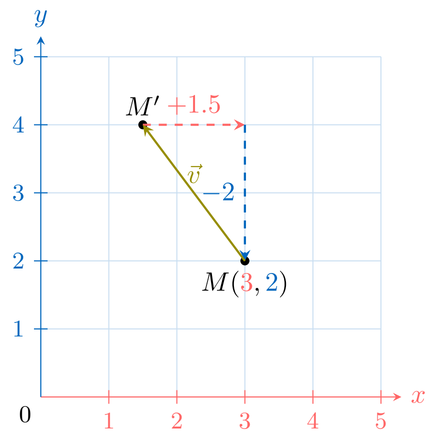
Ex 6: Find the coordinates of the point M whose image is $M'(1.5,4)$ under a translation by vector $\vec{v}\begin{pmatrix}-1.5\\2\end{pmatrix}$.

$$M(\boxed{3},\boxed{2})$$

Answer:

$$\overrightarrow{MM'}=\vec{v}$$
$$\begin{pmatrix}1.5-x\\4-y\end{pmatrix}=\begin{pmatrix}-1.5\\2\end{pmatrix}$$
$$1.5-x=-1.5 \text{ and } 4-y=2$$
$$x=1.5+1.5=3 \text{ and } y=4-2=2$$

so $M(3,2)$



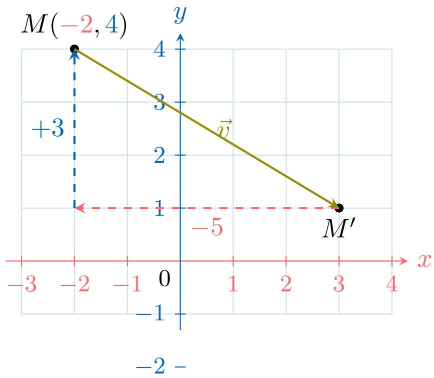
Ex 7: Find the coordinates of the point M whose image is $M'(3,1)$ under a translation by vector $\vec{v}\begin{pmatrix}5\\-3\end{pmatrix}$.

$$M(\boxed{-2},\boxed{4})$$

Answer:

$$\overrightarrow{MM'}=\vec{v}$$
$$\begin{pmatrix}3-x\\1-y\end{pmatrix}=\begin{pmatrix}5\\-3\end{pmatrix}$$
$$3-x=5 \text{ and } 1-y=-3$$
$$x=3-5=-2 \text{ and } y=1+3=4$$

so $M(-2,4)$



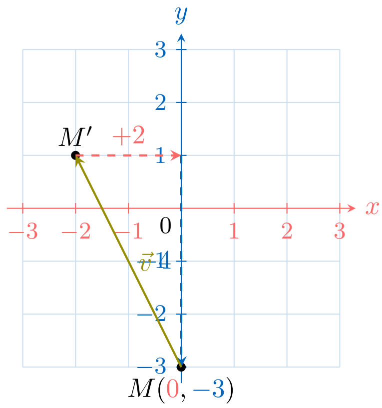
Ex 8: Find the coordinates of the point M whose image is $M'(-2,1)$ under a translation by vector $\vec{v}\begin{pmatrix}-2\\4\end{pmatrix}$.

$$M(\boxed{0},\boxed{-3})$$

Answer:

$$\overrightarrow{MM'}=\vec{v}$$
$$\begin{pmatrix}-2-x\\1-y\end{pmatrix}=\begin{pmatrix}-2\\4\end{pmatrix}$$
$$-2-x=-2 \text{ and } 1-y=4$$
$$x=0 \text{ and } y=1-4=-3$$

so $M(0,-3)$



A.3 DETERMINING THE IMAGE OF LINEAR EQUATION UNDER A TRANSLATION

Ex 9: Find the image equation of $y=2x+1$ under a translation by vector $\begin{pmatrix}1\\-1\end{pmatrix}$.

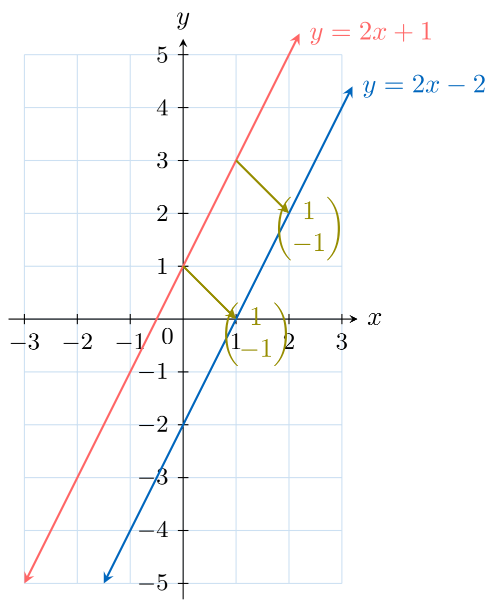
$$y=\boxed{2x-2}$$

Answer: The transformation equations are:

$$x'=x-1 \quad\text{and}\quad y'=y+2$$
$$x=x'+1 \quad\text{and}\quad y=y'-2$$

To find the image equation under translation by vector $\begin{pmatrix}1\\-1\end{pmatrix}$, replace x by $x'-1$ and y by $y'+1$ in the original equation:

$$y'+1=2(x'-1)+1$$
$$y'+1=2x'-2+1$$
$$y'+1=2x'-1$$
$$y'=2x'-2$$



Ex 10: Find the image equation of $y = -x + 1$ under a translation by vector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

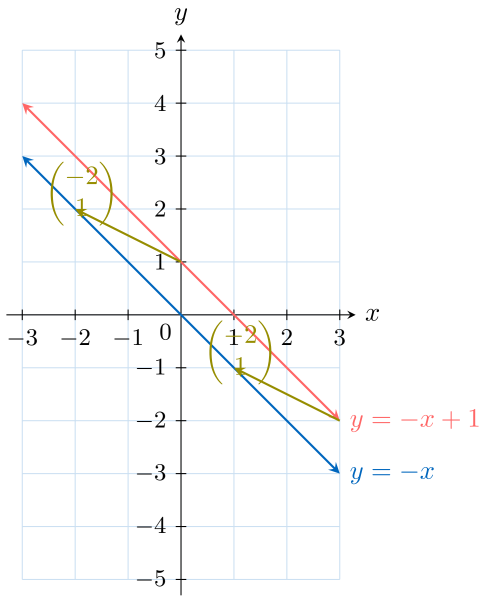
$$y = \boxed{-x}$$

Answer: The transformation equations are:

$$x' = x - 2 \quad \text{and} \quad y' = y + 1$$
$$x = x' + 2 \quad \text{and} \quad y = y' - 1$$

To find the image equation under translation by vector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, replace x by $x' + 2$ and y by $y' - 1$ in the original equation:

$$y' - 1 = -(x' + 2) + 1$$
$$y' - 1 = -x' - 2 + 1$$
$$y' - 1 = -x' - 1$$
$$y' = -x'$$



Ex 11: Find the image equation of $y = \frac{x}{2}$ under a translation by vector $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$.

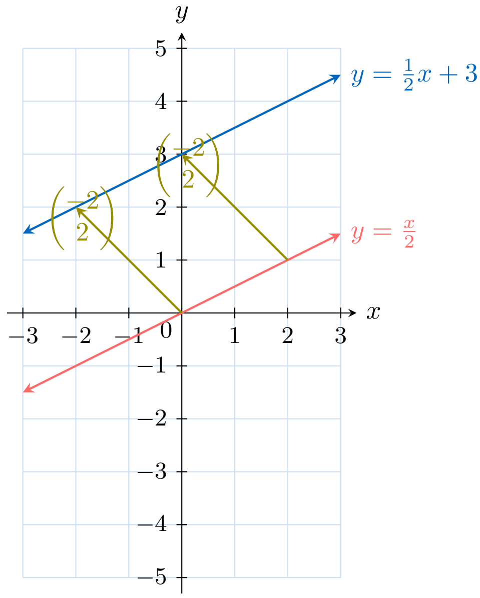
$$y = \boxed{\frac{1}{2}x + 3}$$

Answer: The transformation equations are:

$$x' = x - 2 \quad \text{and} \quad y' = y + 2$$
$$x = x' + 2 \quad \text{and} \quad y = y' - 2$$

To find the image equation under translation by vector $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$, replace x by $x' + 2$ and y by $y' - 2$ in the original equation:

$$y' - 2 = \frac{1}{2}(x' + 2)$$
$$y' - 2 = \frac{1}{2}x' + 1$$
$$y' = \frac{1}{2}x' + 3$$



B HOMOTHETY

B.1 DETERMINING THE IMAGE UNDER A HOMOTHETY

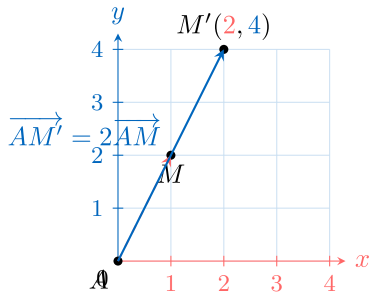
Ex 12: Find the coordinates of the image point M' of point $M(1,2)$ under a homothety with center $A(0,0)$ and scale factor 2.

$$M'(\boxed{2},\boxed{4})$$

Answer:

$$\overrightarrow{AM'} = 2\overrightarrow{AM}$$
$$\begin{pmatrix} x' - 0 \\ y' - 0 \end{pmatrix} = 2\begin{pmatrix} 1 - 0 \\ 2 - 0 \end{pmatrix}$$
$$x' = 2(1) \text{ and } y' = 2(2)$$
$$x' = 2 \text{ and } y' = 4$$

so $M'(2,4)$



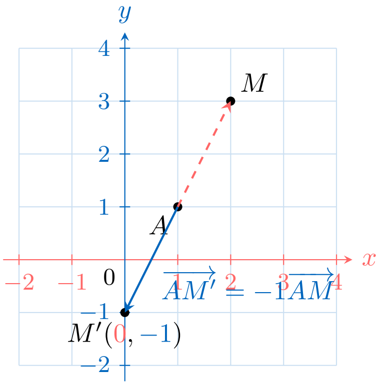
Ex 13: Find the coordinates of the image point M' of point $M(2,3)$ under a homothety with center $A(1,1)$ and scale factor -1.

$$M'(\boxed{0},\boxed{-1})$$

Answer:

$$\overrightarrow{AM'} = -1\overrightarrow{AM}$$
$$\begin{pmatrix} x' - 1 \\ y' - 1 \end{pmatrix} = -1\begin{pmatrix} 2 - 1 \\ 3 - 1 \end{pmatrix}$$
$$x' - 1 = -1(1) \text{ and } y' - 1 = -1(2)$$
$$x' - 1 = -1 \text{ and } y' - 1 = -2$$
$$x' = 0 \text{ and } y' = -1$$

so $M'(0,-1)$



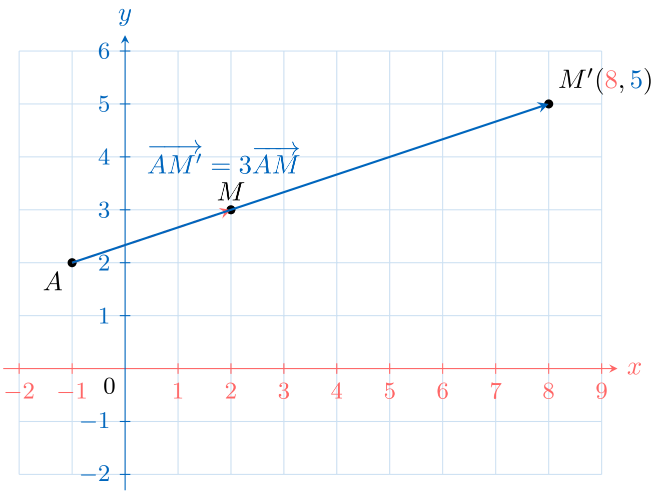
Ex 14: Find the coordinates of the image point M' of point $M(2,3)$ under a homothety with center $A(-1,2)$ and scale factor 3.

$$M'(\boxed{8},\boxed{5})$$

Answer:

$$\overrightarrow{AM'} = 3\overrightarrow{AM}$$
$$\begin{pmatrix} x'-(-1) \\ y'-2 \end{pmatrix} = 3\begin{pmatrix} 2-(-1) \\ 3-2 \end{pmatrix}$$
$$x'+1 = 3(3) \text{ and } y'-2 = 3(1)$$
$$x'+1 = 9 \text{ and } y'-2 = 3$$
$$x' = 8 \text{ and } y' = 5$$

so $M'(8,5)$



B.2 DETERMINING THE ORIGINAL POINT UNDER A HOMOTHETY

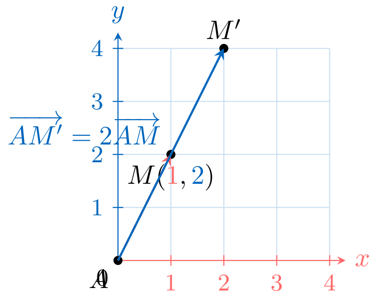
Ex 15: Find the coordinates of the point M whose image is $M'(2,4)$ under a homothety with center $A(0,0)$ and scale factor 2.

$$M(\boxed{1},\boxed{2})$$

Answer:

$$\overrightarrow{AM'} = 2\overrightarrow{AM}$$
$$\begin{pmatrix} 2-0 \\ 4-0 \end{pmatrix} = 2\begin{pmatrix} x-0 \\ y-0 \end{pmatrix}$$
$$2 = 2x \text{ and } 4 = 2y$$
$$x = 2/2 \text{ and } y = 4/2$$
$$x = 1 \text{ and } y = 2$$

so $M(1,2)$



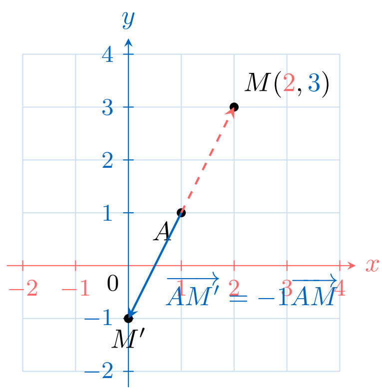
Ex 16: Find the coordinates of the point M whose image is $M'(0,-1)$ under a homothety with center $A(1,1)$ and scale factor -1.

$$M(\boxed{2},\boxed{3})$$

Answer:

$$\overrightarrow{AM'} = -1\overrightarrow{AM}$$
$$\begin{pmatrix} 0-1 \\ -1-1 \end{pmatrix} = -1\begin{pmatrix} x-1 \\ y-1 \end{pmatrix}$$
$$-1 = -1(x-1) \text{ and } -2 = -1(y-1)$$
$$-1 = -x+1 \text{ and } -2 = -y+1$$
$$x-1 = 1 \text{ and } y-1 = 2$$
$$x = 2 \text{ and } y = 3$$

so $M(2,3)$



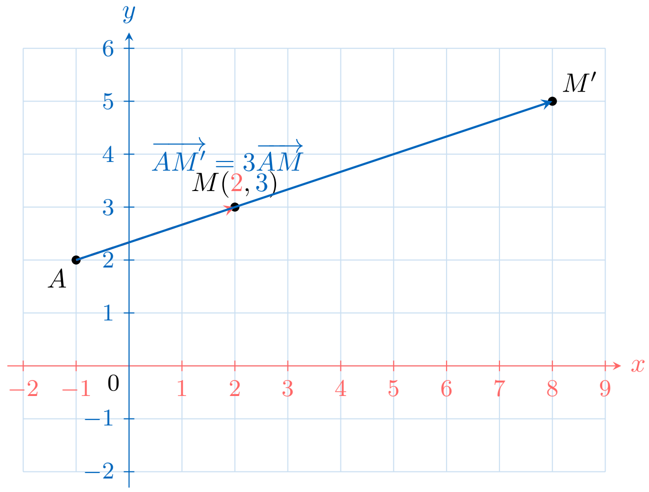
Ex 17: Find the coordinates of the point M whose image is $M'(8,5)$ under a homothety with center $A(-1,2)$ and scale factor 3.

$$M(\boxed{2},\boxed{3})$$

Answer:

$$\overrightarrow{AM'} = 3\overrightarrow{AM}$$
$$\begin{pmatrix} 8-(-1) \\ 5-2 \end{pmatrix} = 3\begin{pmatrix} x-(-1) \\ y-2 \end{pmatrix}$$
$$9 = 3(x+1) \text{ and } 3 = 3(y-2)$$
$$9/3 = x+1 \text{ and } 3/3 = y-2$$
$$x+1 = 3 \text{ and } y-2 = 1$$
$$x = 2 \text{ and } y = 3$$

so $M(2,3)$



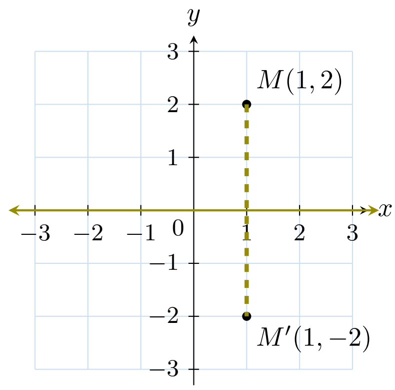
C SPECIFIC REFLECTIONS

C.1 DETERMINING THE IMAGE UNDER A REFLECTION

Ex 18: Find the coordinates of the image point M' of point $M(1,2)$ under reflection over the x-axis.

$M'(\boxed{1},\boxed{-2})$

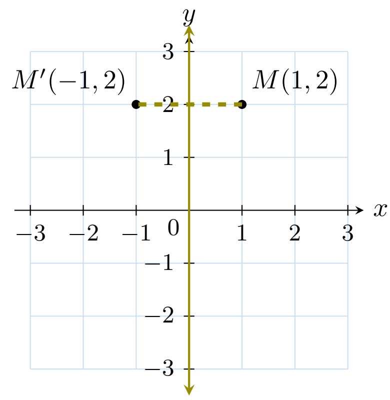
Answer: The image of $M(x,y)$ under reflection over the x-axis is $M'(x,-y)$.
So, for $M(1,2)$, $M'(1,-2)$.



Ex 19: Find the coordinates of the image point M' of point $M(1,2)$ under reflection over the y-axis.

$M'(\boxed{-1},\boxed{2})$

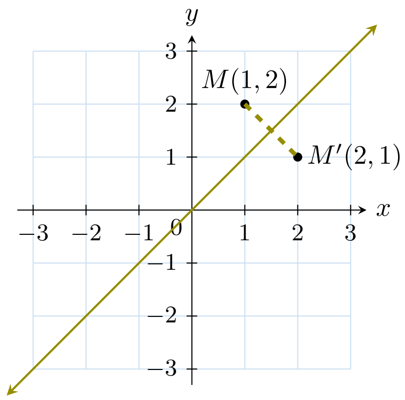
Answer: The image of $M(x,y)$ under reflection over the y-axis is $M'(-x,y)$.
So, for $M(1,2)$, $M'(-1,2)$.



Ex 20: Find the coordinates of the image point M' of point $M(1,2)$ under reflection over the line $y=x$.

$M'(\boxed{2},\boxed{1})$

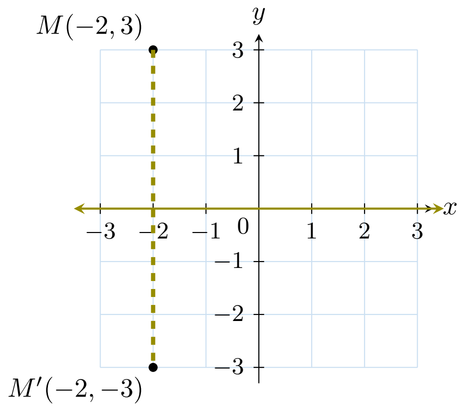
Answer: The image of $M(x,y)$ under reflection over the line $y=x$ is $M'(y,x)$.
So, for $M(1,2)$, $M'(2,1)$.



Ex 21: Find the coordinates of the image point M' of point $M(-2,3)$ under reflection over the x-axis.

$M'(\boxed{-2},\boxed{-3})$

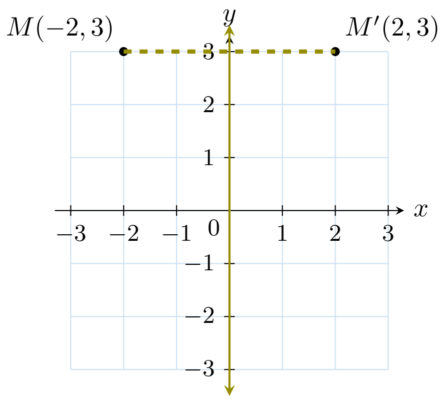
Answer: The image of $M(x,y)$ under reflection over the x-axis is $M'(x,-y)$.
So, for $M(-2,3)$, $M'(-2,-3)$.



Ex 22: Find the coordinates of the image point M' of point $M(-2,3)$ under reflection over the y-axis.

$M'(\boxed{2},\boxed{3})$

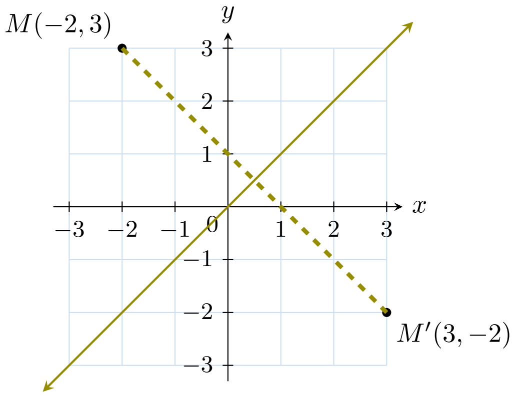
Answer: The image of $M(x,y)$ under reflection over the y-axis is $M'(-x,y)$.
So, for $M(-2,3)$, $M'(2,3)$.



Ex 23: Find the coordinates of the image point M' of point $M(-2, 3)$ under reflection over the line $y = x$.

$$M'(\boxed{3}, \boxed{-2})$$

Answer: The image of $M(x, y)$ under reflection over the line $y = x$ is $M'(y, x)$.
So, for $M(-2, 3)$, $M'(3, -2)$.



C.2 DETERMINING THE IMAGE OF LINEAR EQUATION UNDER A REFLECTION

Ex 24: Find the image equation of $y = 2x + 1$ under a reflection over the x-axis.

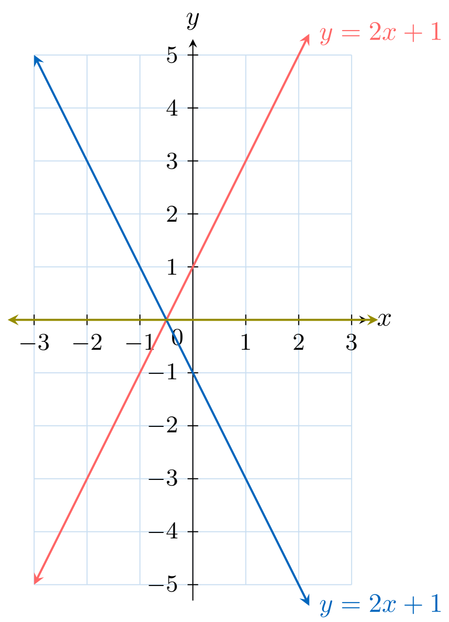
$$y = \boxed{-2x - 1}$$

Answer: To find the image equation under reflection over the x-axis, replace y by $-y$ in the original equation:

$$-y = 2x + 1$$

Multiply both sides by -1:

$$y = -2x - 1$$



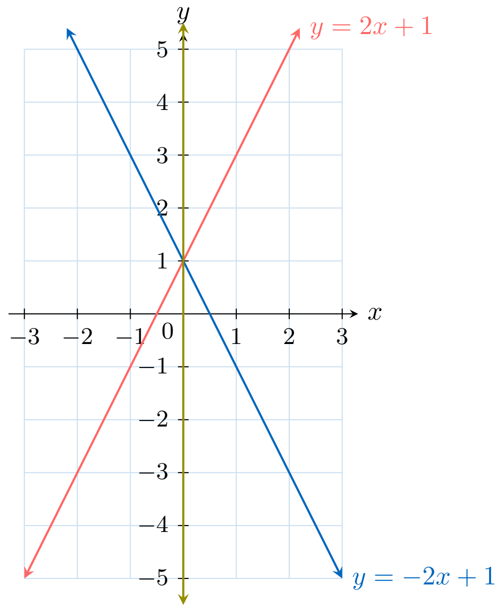
Ex 25: Find the image equation of $y = 2x + 1$ under a reflection over the y-axis.

$$y = \boxed{-2x + 1}$$

Answer: To find the image equation under reflection over the y-axis, replace x by $-x$ in the original equation:

$$y = 2(-x) + 1$$

$$y = -2x + 1$$



Ex 26: Find the image equation of $y = 2x + 1$ under a reflection over the line $y = x$.

$$y = \boxed{\frac{1}{2}x - \frac{1}{2}}$$

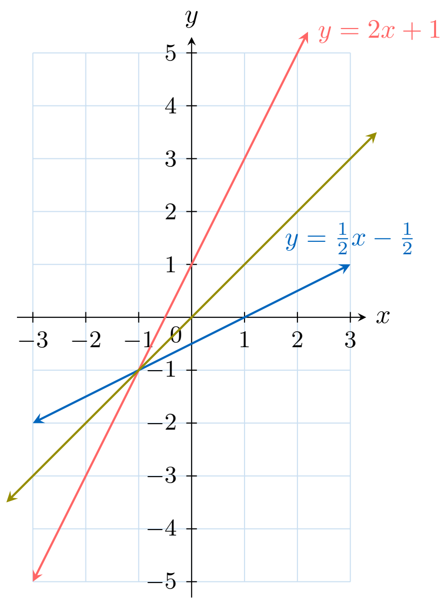
Answer: To find the image equation under reflection over the line $y = x$, replace x by y and y by x in the original equation:

$$x = 2y + 1$$

Solve for y:

$$x - 1 = 2y$$

$$y = \frac{x - 1}{2} = \frac{1}{2}x - \frac{1}{2}$$



Ex 27: Find the image equation of $y = \frac{x}{2} - 1$ under a reflection over the x-axis.

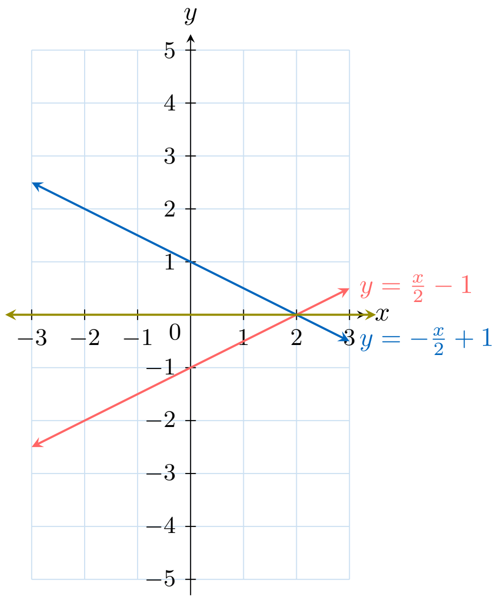
$$y = \boxed{-\frac{x}{2}+1}$$

Answer: To find the image equation under reflection over the x-axis, replace y by $-y$ in the original equation:

$$-y = \frac{x}{2} - 1$$

Multiply both sides by -1:

$$y = -\frac{x}{2} + 1$$



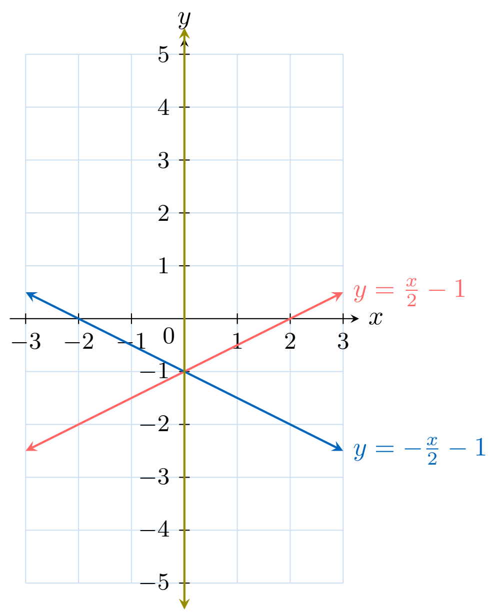
Ex 28: Find the image equation of $y = \frac{x}{2} - 1$ under a reflection over the y-axis.

$$y = \boxed{-\frac{x}{2}-1}$$

Answer: To find the image equation under reflection over the y-axis, replace x by $-x$ in the original equation:

$$y = \frac{-x}{2} - 1$$

$$y = -\frac{x}{2} - 1$$



Ex 29: Find the image equation of $y = \frac{x}{2} - 1$ under a reflection over the line $y = x$.

$$y = \boxed{2x+2}$$

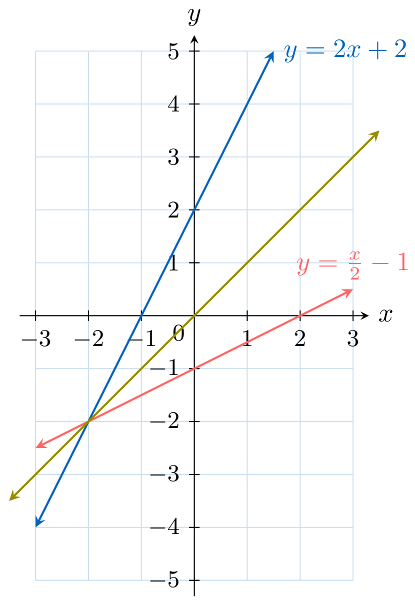
Answer: To find the image equation under reflection over the line $y = x$, replace x by y and y by x in the original equation:

$$x = \frac{y}{2} - 1$$

Solve for y:

$$x + 1 = \frac{y}{2}$$

$$y = 2(x+1) = 2x + 2$$



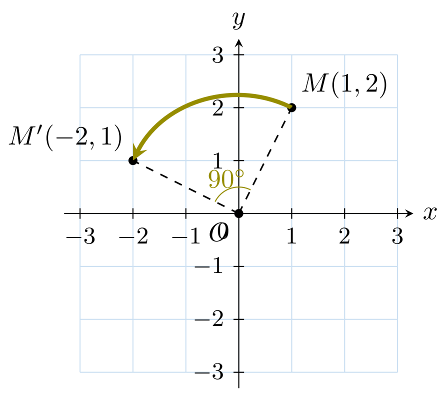
D SPECIFIC ROTATIONS

D.1 DETERMINING THE IMAGE UNDER A ROTATION

Ex 30: Find the coordinates of the image point M' of point $M(1,2)$ under a rotation of 90° (counterclockwise) around the origin.

$$M'(\boxed{-2}, \boxed{1})$$

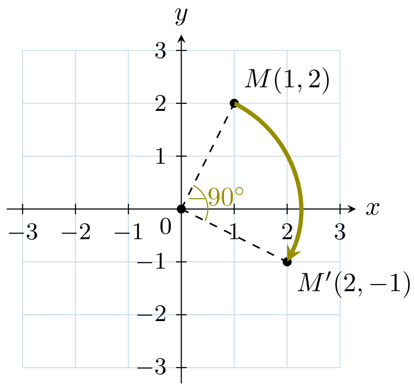
Answer: The image of $M(x, y)$ under a rotation of 90° (counterclockwise) around the origin is $M'(-y, x)$.
So, for $M(1,2)$, $M'(-2,1)$.



Ex 31: Find the coordinates of the image point M' of point $M(1,2)$ under a rotation of -90° (clockwise) around the origin.

$$M'(\boxed{2},\boxed{-1})$$

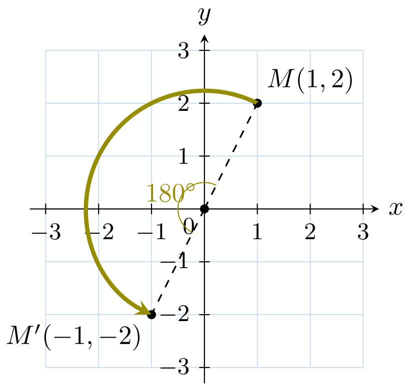
Answer: The image of $M(x, y)$ under a rotation of -90° (clockwise) around the origin is $M'(y, -x)$.
So, for $M(1, 2)$, $M'(2, -1)$.



Ex 32: Find the coordinates of the image point M' of point $M(1, 2)$ under a rotation of 180° around the origin.

$$M'(\boxed{-1},\boxed{-2})$$

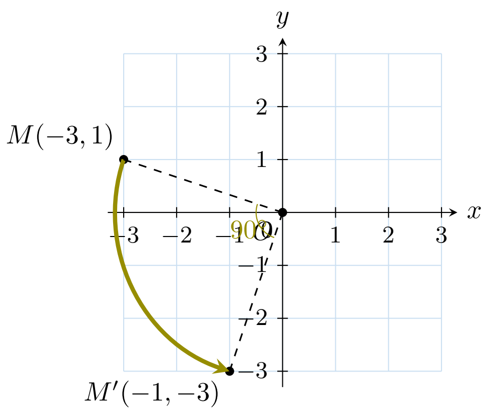
Answer: The image of $M(x, y)$ under a rotation of 180° around the origin is $M'(-x, -y)$.
So, for $M(1, 2)$, $M'(-1, -2)$.



Ex 33: Find the coordinates of the image point M' of point $M(-3, 1)$ under a rotation of 90° (counterclockwise) around the origin.

$$M'(\boxed{-1},\boxed{-3})$$

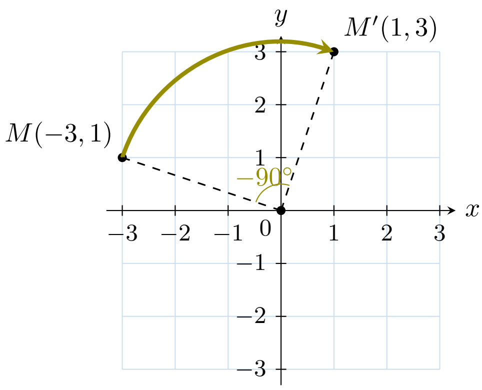
Answer: The image of $M(x, y)$ under a rotation of 90° (counterclockwise) around the origin is $M'(-y, x)$.
So, for $M(-3, 1)$, $M'(-1, -3)$.



Ex 34: Find the coordinates of the image point M' of point $M(-3, 1)$ under a rotation of -90° (clockwise) around the origin.

$$M'(\boxed{1},\boxed{3})$$

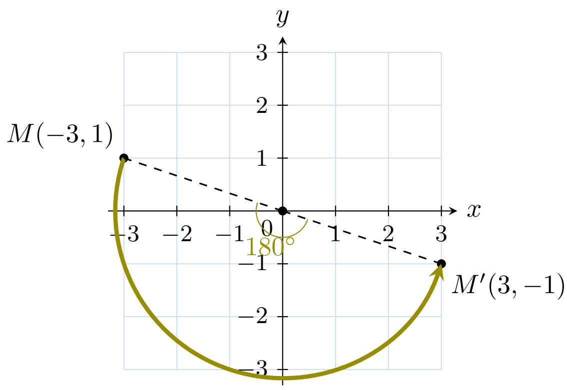
Answer: The image of $M(x, y)$ under a rotation of -90° (clockwise) around the origin is $M'(y, -x)$.
So, for $M(-3, 1)$, $M'(1, 3)$.



Ex 35: Find the coordinates of the image point M' of point $M(-3, 1)$ under a rotation of 180° around the origin.

$$M'(\boxed{3},\boxed{-1})$$

Answer: The image of $M(x, y)$ under a rotation of 180° around the origin is $M'(-x, -y)$.
So, for $M(-3, 1)$, $M'(3, -1)$.



D.2 DETERMINING THE IMAGE OF LINEAR EQUATION UNDER A ROTATION

Ex 36: Find the image equation of $y = 2x + 1$ under a rotation of 90° (counterclockwise) around the origin.

$$y = \boxed{-\frac{1}{2}x - \frac{1}{2}}$$

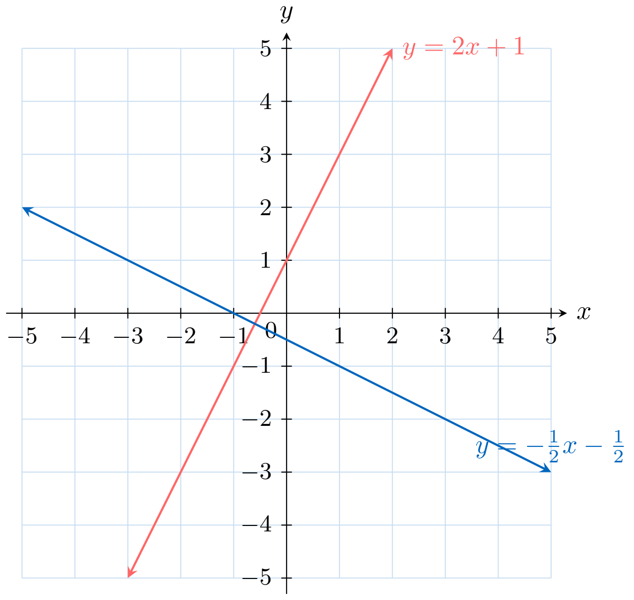
Answer: To find the image equation under a rotation of 90° (counterclockwise) around the origin, replace x by y and y by $-x$ in the original equation (since the inverse mapping is used for the equation):

$$-x = 2y + 1$$

Solve for y:

$$-x - 1 = 2y$$

$$y = \frac{-x - 1}{2} = -\frac{1}{2}x - \frac{1}{2}$$



Ex 37: Find the image equation of $y = 2x + 1$ under a rotation of -90° (clockwise) around the origin.

$$y = \boxed{-\frac{1}{2}x + \frac{1}{2}}$$

Answer: To find the image equation under a rotation of -90° (clockwise) around the origin, replace x by $-y$ and y by x in the original equation:

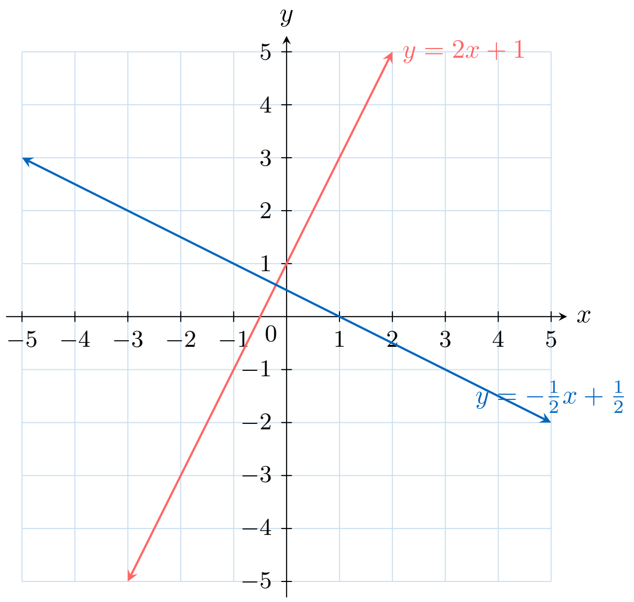
$$x = 2(-y) + 1$$

$$x = -2y + 1$$

Solve for y:

$$x - 1 = -2y$$

$$y = \frac{1 - x}{2} = -\frac{1}{2}x + \frac{1}{2}$$



Ex 38: Find the image equation of $y = 2x + 1$ under a rotation of 180° around the origin.

$$y = \boxed{2x - 1}$$

Answer: To find the image equation under a rotation of 180° around the origin, replace x by $-x$ and y by $-y$ in the original equation:

$$-y = 2(-x) + 1$$

$$-y = -2x + 1$$

Multiply both sides by -1:

$$y = 2x - 1$$