TRANSFORMATIONS

A TYPES OF TRANSFORMATIONS

Transformations are ways to move, flip, or turn a shape.

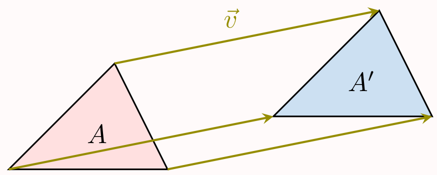
Definition Object and Image

When a transformation is applied to a shape, the input shape is called the **object**. The output shape after the transformation is called the **image**.

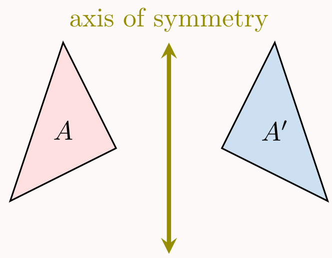
Definition Types of Transformations

There are several types of transformations, including:

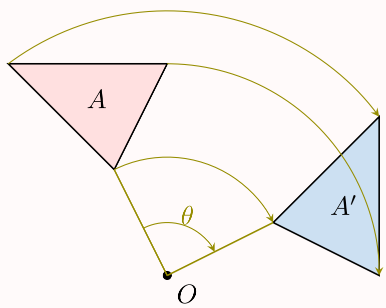
- **Translation:** Slides the shape to a new position without changing its shape, size, or the direction it faces.



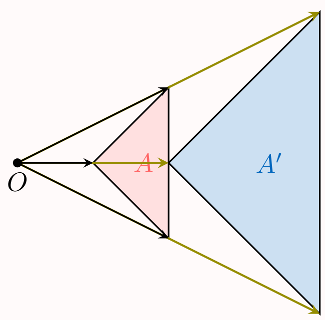
- **Reflection:** Flips the shape over a line (like a mirror), creating a mirror image.



- **Rotation:** Turns the shape around a point by a certain angle.



- **Homothety:** Enlarges or reduces the shape by a scale factor from a center point, keeping the shape similar.

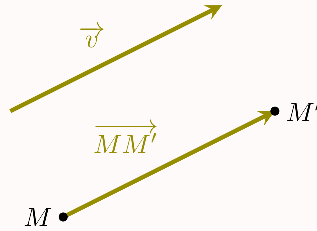


B TRANSLATION

A **translation** moves a figure from one place to another. Every point on the figure moves the same distance in the same direction.

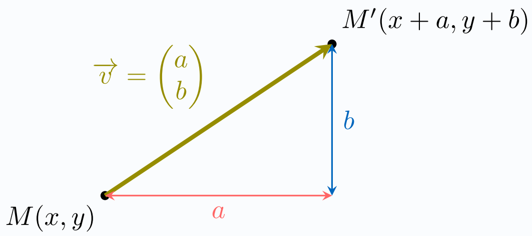
Definition Translation

A translation by the vector $\vec{v}$ maps a point M to its image M' such that $\overrightarrow{MM'} = \vec{v}$.



Proposition Coordinates of the Image Point

In a coordinate system, if the point M has coordinates (x, y) and the translation vector is $\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$, then the image point M' has coordinates $(x+a, y+b)$.



Proof

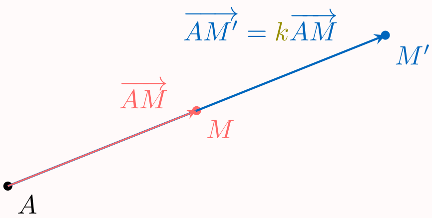
Let M' be the image point with coordinates (x', y').

$$\overrightarrow{MM'} = \vec{v}$$
$$\begin{pmatrix} x'-x \\ y'-y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$
$$x' - x = a \text{ and } y' - y = b$$
$$x' = x + a \text{ and } y' = y + b$$

C HOMOTHETY

Definition Homothety

A **homothety** with center A and scale factor k maps a point M to the point M' such that M' is obtained by translating the point A by the vector $k\overrightarrow{AM}$.



$$\frac{AM'}{AM} = |k|$$

Proposition Coordinates of the Image Point

In a coordinate system, if the center A has coordinates (a, b), the point M has coordinates (x, y), and the scale factor is k, then the image point M' has coordinates $(a + k(x-a),\ b + k(y-b))$.

Proof

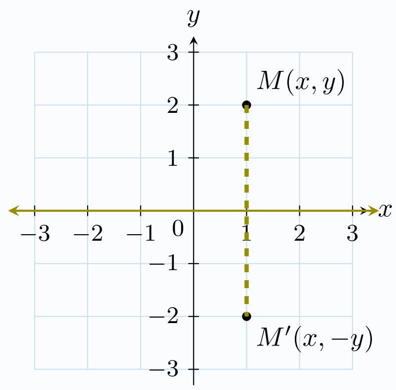
Let M' be the image point with coordinates (x', y').

$$\overrightarrow{AM'} = k\overrightarrow{AM}$$
$$\begin{pmatrix} x'-a \\ y'-b \end{pmatrix} = k\begin{pmatrix} x-a \\ y-b \end{pmatrix}$$
$$x' - a = k(x-a) \text{ and } y' - b = k(y-b)$$
$$x' = a + k(x-a) \text{ and } y' = b + k(y-b)$$

D SPECIFIC REFLECTIONS

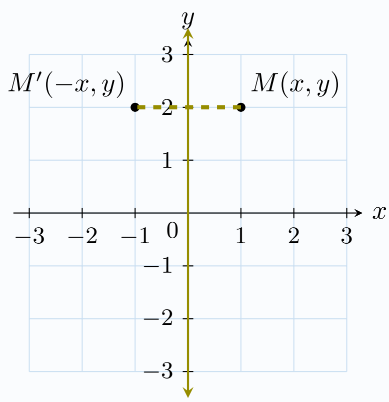
Proposition Reflection over the x-axis

The image of the point $M(x, y)$ under the reflection over the x-axis is $M'(x, -y)$.



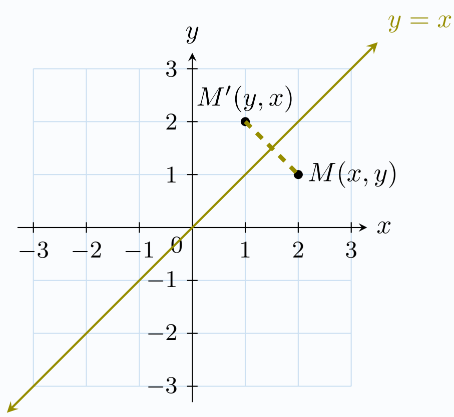
Proposition Reflection over the y-axis

The image of the point $M(x, y)$ under the reflection over the y-axis is $M'(-x, y)$.



Proposition Reflection over the line $y = x$

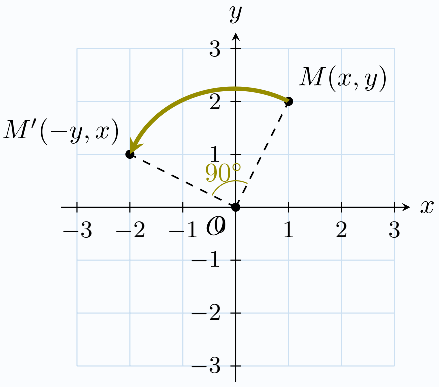
The image of the point $M(x, y)$ under the reflection $M_{y=x}$ over the line $y = x$ is $M'(y, x)$.



E SPECIFIC ROTATIONS

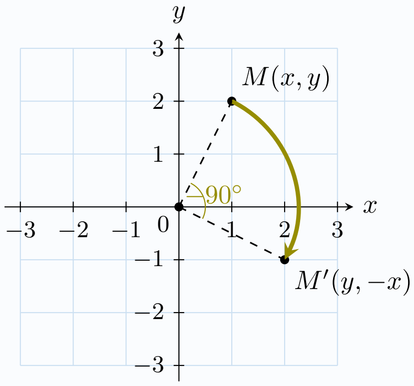
Proposition Rotation of 90°

The image of the point $M(x,y)$ under the rotation of 90° (counterclockwise) around the origin is $M'(-y,x)$.



Proposition Rotation of -90°

The image of the point $M(x,y)$ under the rotation of -90° (clockwise) around the origin is $M'(y,-x)$.



Proposition Rotation of 180°

The image of the point $M(x,y)$ under the rotation of 180° around the origin is $M'(-x,-y)$.