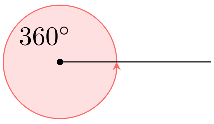
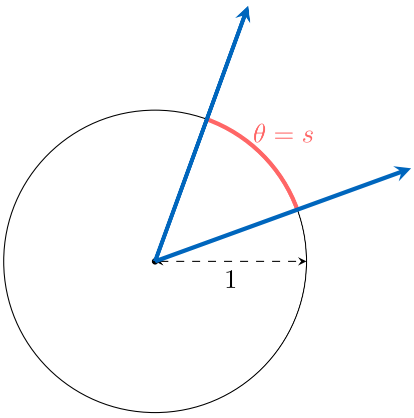
TRIGONOMETRY

- The measure of an angle is the fraction of a complete turn. We have seen previously that a full revolution makes an angle of 360°.



Although a full revolution could just as well be 100°, or 120°, or some other number of degrees. This makes degrees a rather artificial measure of angles.

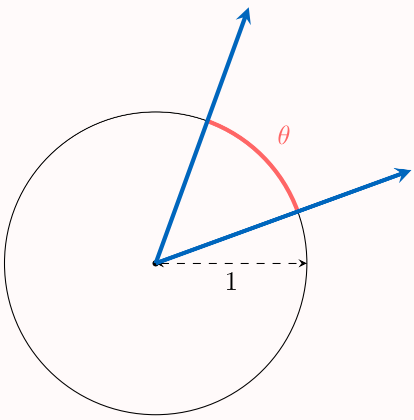
- An angle can be equal to the length of the arc formed by an angle with the circle of radius 1.



A RADIAN MEASURE

Definition Radian Measure

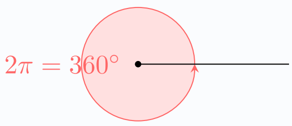
The **radian measure**, θ, of an angle is defined as the length of the arc subtended between the two rays of the angle on a circle of radius 1.

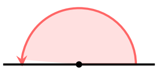


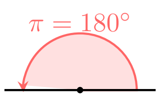
The radian unit is often omitted with 1 rad = 1.

Proposition Angle of Complete Turn

The radian measure of the revolution angle is 2π.



Ex: Find the angle of the straight angle 

Answer: The straight angle is half the revolution angle. So the angle is $\frac{2\pi}{2} = \pi$. 

Proposition **Proportionality of Radians and Degrees**

Radians and degrees are in proportion.

Method **Converting Between Degrees and Radians**

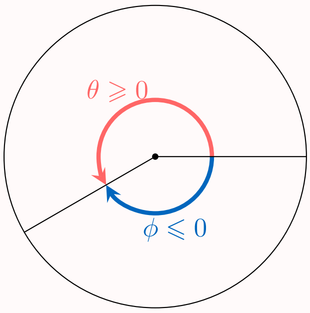
- To convert from degrees to radians, multiply degrees by $\frac{\pi}{180}$
- To convert from radians to degrees, multiply radians by $\frac{180}{\pi}$

Ex: Convert 60° to radians.

Answer: $$\begin{aligned} 60^\circ &= 60^\circ \times \frac{\pi}{180^\circ} \\ &= \frac{\pi}{3} \end{aligned}$$

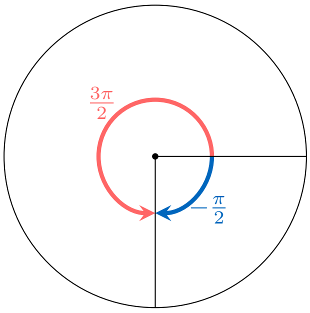
Definition **Positive and Negative Angles**

- A **positive angle measure** represents a **counterclockwise rotation**.
- A **negative angle measure** represents a **clockwise rotation**.

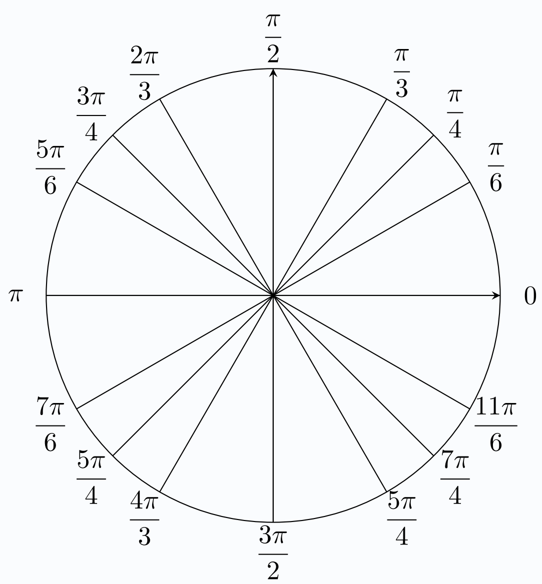


Ex: Draw the angle $\frac{3\pi}{2}$ and $-\frac{\pi}{2}$

Answer:



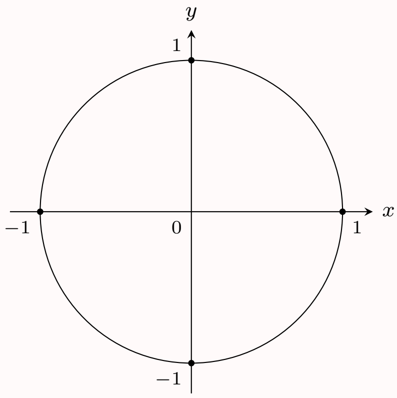
Proposition **Reference Angles on the Unit Circle**



B TRIGONOMETRIC RATIOS AND UNIT CIRCLE

Definition Unit circle

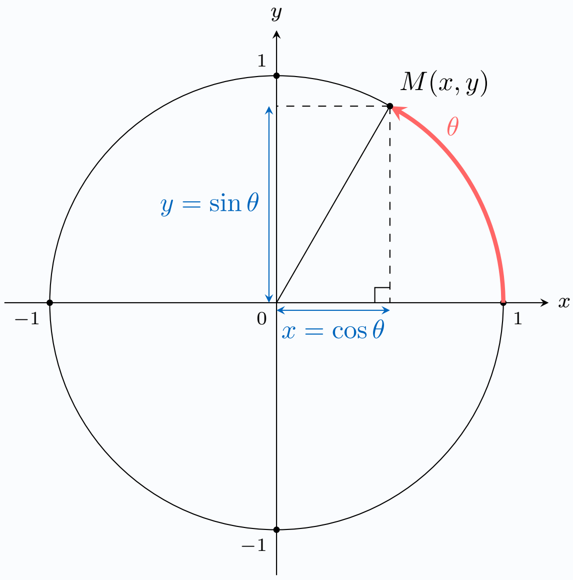
The **unit circle** is a circle with a radius of 1 centered at the origin.



Proposition Relation between Angle and Coordinates

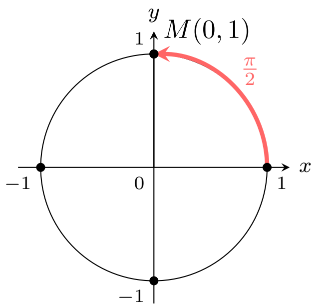
For $M(x, y)$ the point on the unit circle at the angle θ, then

- $\cos\theta$ is the x-coordinate of M: $\cos\theta = x$
- $\sin\theta$ is the y-coordinate of M: $\sin\theta = y$



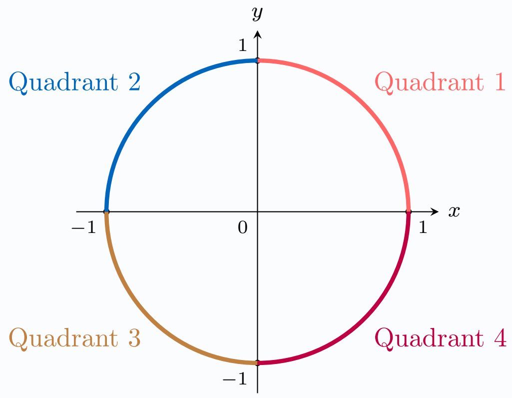
Ex: Find the values $\cos\left(\frac{\pi}{2}\right)$ and $\sin\left(\frac{\pi}{2}\right)$.

Answer: On the unit circle, the point corresponding to the angle $\frac{\pi}{2}$ has coordinates (0,1):



$\cos\left(\frac{\pi}{2}\right) = 0$ x-coordinate
$\sin\left(\frac{\pi}{2}\right) = 1$ y-coordinate

Proposition Sign of of Sine and Cosine



Quadrant	$\cos\theta$	$\sin\theta$
1	$+$	$+$
2	$-$	$+$
3	$-$	$-$
4	$+$	$-$

C TRIGONOMETRIC PROPERTIES

Proposition Pythagorean Relation

$$\cos^2\theta + \sin^2\theta = 1$$

Proposition Maximum and Minimum of Trigonometric Ratios

$$-1 \leqslant \cos\theta \leqslant 1 \quad \text{and} \quad -1 \leqslant \sin\theta \leqslant 1$$

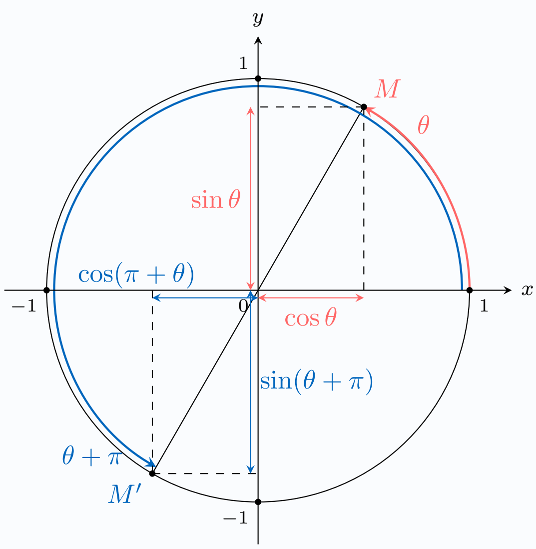
Proposition Periodicity of Trigonometric Ratios

$$\cos(\theta + 2k\pi) = \cos\theta \quad \text{and} \quad \sin(\theta + 2k\pi) = \sin\theta$$

where k is any integer.

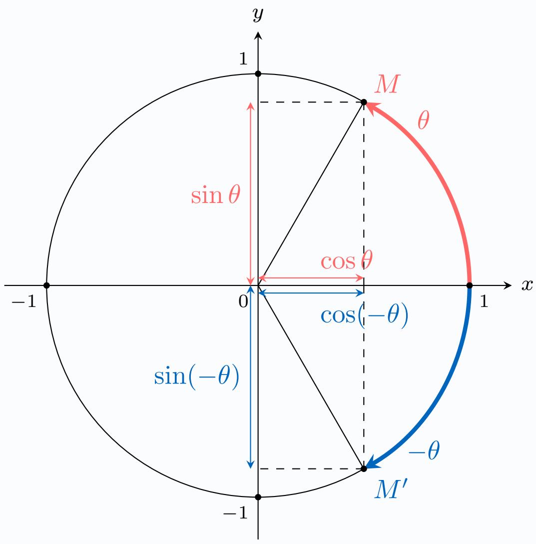
Proposition Add π to Trigonometric Ratios

$$\sin(\pi + \theta) = -\sin\theta$$
$$\cos(\pi + \theta) = -\cos\theta$$



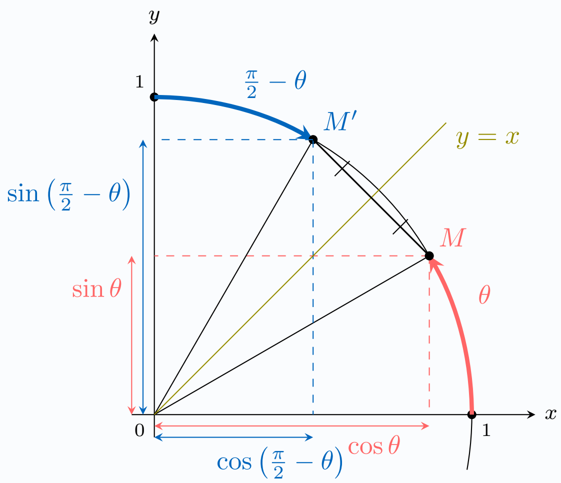
Proposition Opposite of Trigonometric Ratios

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$



Proposition Identities with $\frac{\pi}{2} - x$

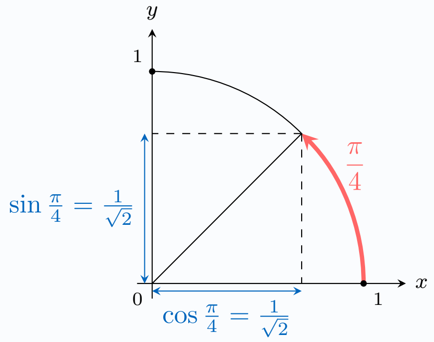
$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$
$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$



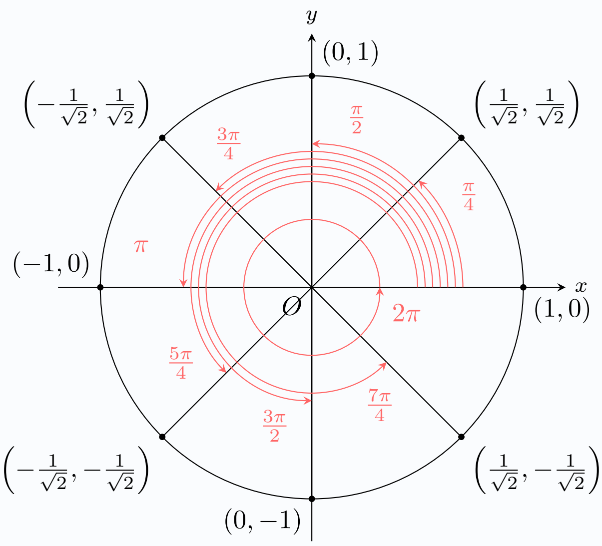
D MULTIPLES OF $\frac{\pi}{4}$

Proposition **Coordinates of Angle $\frac{\pi}{4}$**

$$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \quad \text{and} \quad \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$



Proposition **Multiples of $\frac{\pi}{4}$**



The signs of the coordinates are determined by the quadrant in which the angle lies.

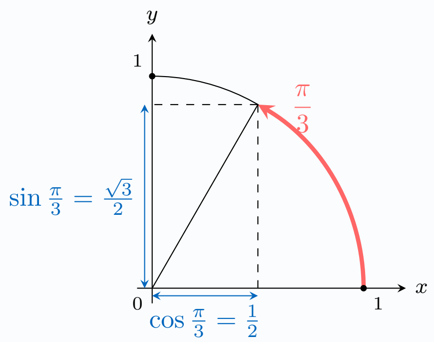
Ex: Find $\cos\frac{3\pi}{4}$

Answer: $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

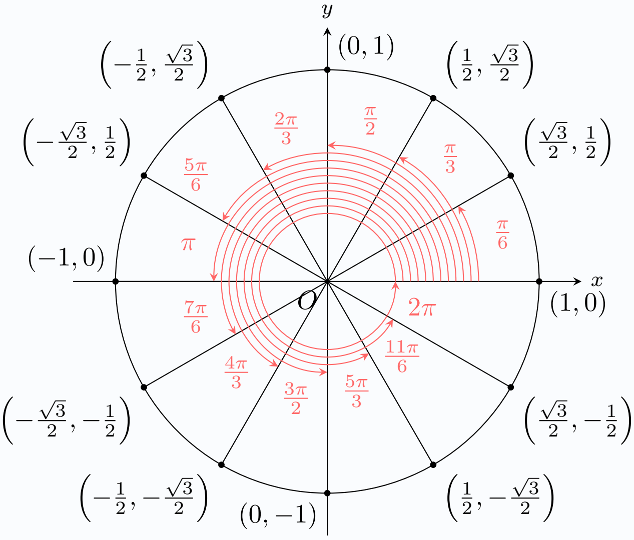
E MULTIPLES OF $\frac{\pi}{6}$

Proposition **Coordinates of Angle $\frac{\pi}{3}$**

$$\cos\frac{\pi}{3} = \frac{1}{2} \quad \text{and} \quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

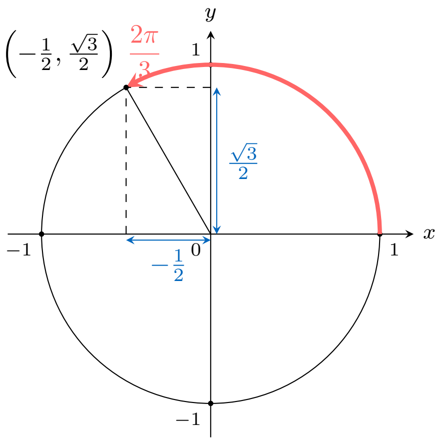


Proposition Multiples of $\frac{\pi}{6}$



Ex: Find $\cos\frac{2\pi}{3}$ and $\sin\frac{2\pi}{3}$

Answer:



$\cos\frac{2\pi}{3} = -\frac{1}{2}$ and $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$