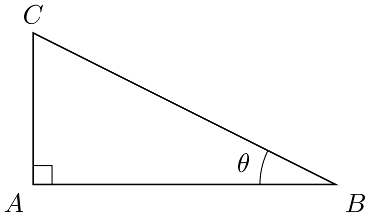
TRIGONOMETRY

A RIGHT-ANGLED TRIANGLE

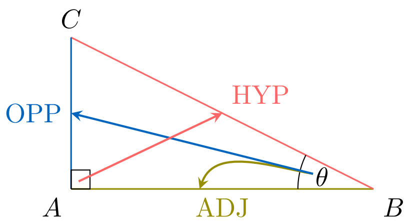
A.1 IDENTIFYING TRIANGLE SIDES

MCQ 1: In the triangle below, identify the adjacent side to the angle θ:



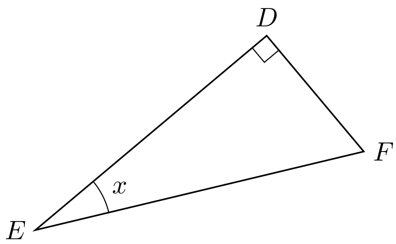
- ☒ $\overline{AB}$
- ☐ $\overline{AC}$
- ☐ $\overline{BC}$

Answer:



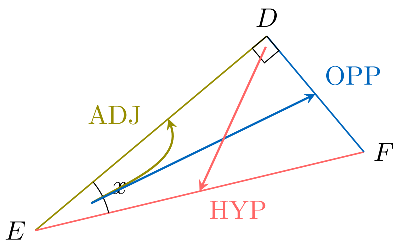
The adjacent side to the angle θ is $\overline{AB}$.

MCQ 2: In the triangle below, identify the hypotenuse relative to the angle x:



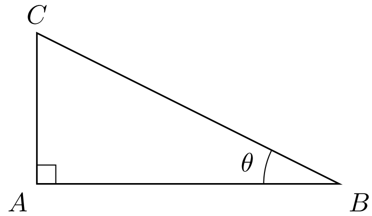
- ☐ $\overline{DE}$
- ☐ $\overline{DF}$
- ☒ $\overline{EF}$

Answer:



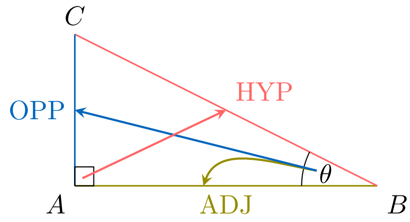
The hypotenuse relative to the angle x is $\overline{EF}$.

MCQ 3: In the triangle below, identify the opposite side to the angle θ:



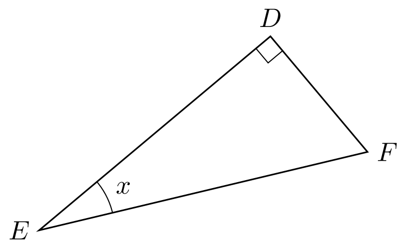
- ☐ $\overline{AB}$
- ☒ $\overline{AC}$
- ☐ $\overline{BC}$

Answer:



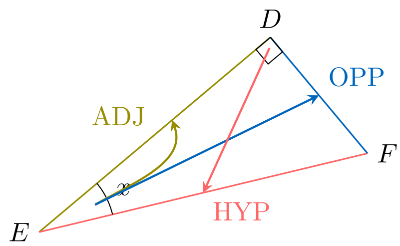
The opposite side to the angle θ is $\overline{AC}$.

MCQ 4: In the triangle below, identify the opposite side to the angle x:



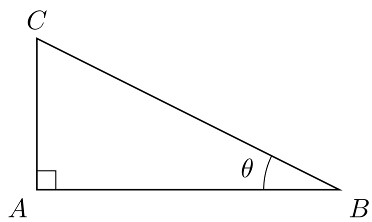
- ☐ $\overline{DE}$
- ☒ $\overline{DF}$
- ☐ $\overline{EF}$

Answer:



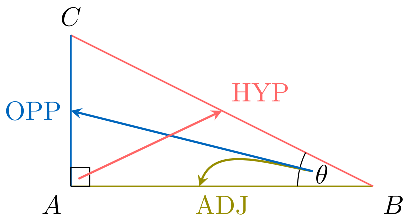
The opposite side to the angle x is $\overline{DF}$.

MCQ 5: In the triangle below, identify the hypotenuse relative to the angle θ:



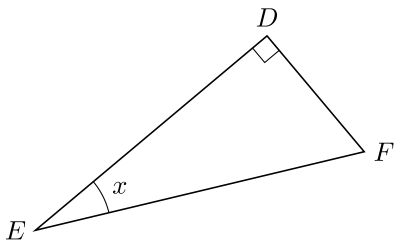
- ☐ $\overline{AB}$
- ☐ $\overline{AC}$
- ☒ $\overline{BC}$

Answer:



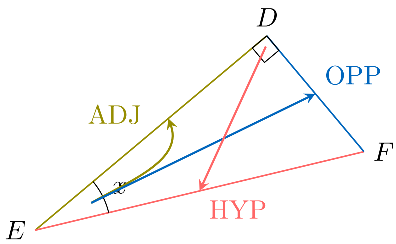
The hypotenuse relative to the angle θ is $\overline{BC}$.

MCQ 6: In the triangle below, identify the adjacent side to the angle x:



- ☒ $\overline{DE}$
- ☐ $\overline{DF}$
- ☐ $\overline{EF}$

Answer:

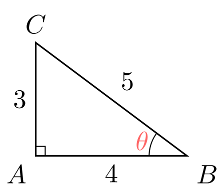


The adjacent side to the angle x is $\overline{DE}$.

B TRIGONOMETRIC FUNCTIONS

B.1 CALCULATING TRIGONOMETRIC RATIOS

Ex 7:



Calculate $\cos(\theta)$.

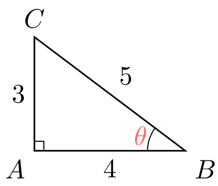
$$\cos(\theta) = \boxed{\frac{4}{5}}$$

Answer: Relative to θ:

- Adjacent side: $AB = 4$
- Hypotenuse: $BC = 5$

$$\begin{aligned}\cos(\theta) &= \frac{\text{ADJ}}{\text{HYP}} \\ &= \frac{4}{5}\end{aligned}$$

Ex 8:



Calculate $\sin(\theta)$.

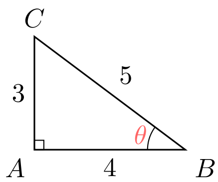
$$\sin(\theta) = \boxed{\frac{3}{5}}$$

Answer: Relative to θ:

- Opposite side: $AC = 3$
- Hypotenuse: $BC = 5$

$$\begin{aligned}\sin(\theta) &= \frac{\text{OPP}}{\text{HYP}} \\ &= \frac{3}{5}\end{aligned}$$

Ex 9:



Calculate $\tan(\theta)$.

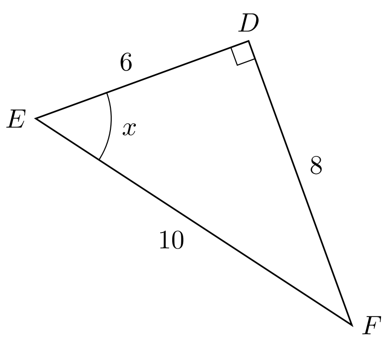
$$\tan(\theta) = \boxed{\frac{3}{4}}$$

Answer: Relative to θ:

- Opposite side: $AC = 3$
- Adjacent side: $AB = 4$

$$\begin{aligned}\tan(\theta) &= \frac{\text{OPP}}{\text{ADJ}} \\ &= \frac{3}{4}\end{aligned}$$

Ex 10:



Calculate $\sin(x)$.

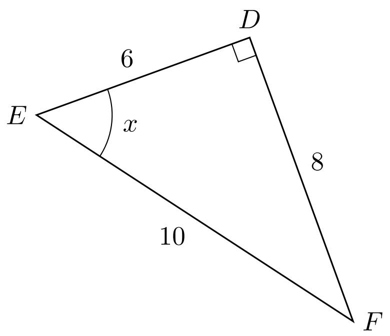
$$\sin(x) = \boxed{\frac{4}{5}}$$

Answer: Relative to x:

- Opposite side: $DF = 8$
- Hypotenuse: $EF = 10$

$$\begin{aligned}\sin(x) &= \frac{\text{OPP}}{\text{HYP}} \\ &= \frac{8}{10} \\ &= \frac{4}{5}\end{aligned}$$

Ex 11:



Calculate $\tan(x)$.

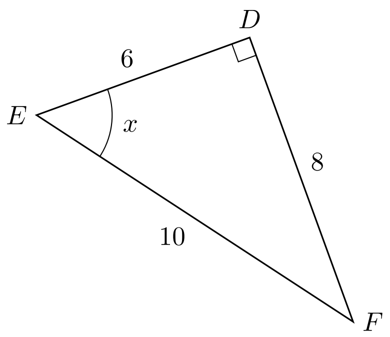
$$\tan(x) = \boxed{\frac{4}{3}}$$

Answer: Relative Fondations of mathematics to x:

- Opposite side: $DF = 8$
- Adjacent side: $DE = 6$

$$\begin{aligned}\tan(x) &= \frac{\text{OPP}}{\text{ADJ}} \\ &= \frac{8}{6} \\ &= \frac{4}{3}\end{aligned}$$

Ex 12:



Calculate $\cos(x)$.

$$\cos(x) = \boxed{\frac{3}{5}}$$

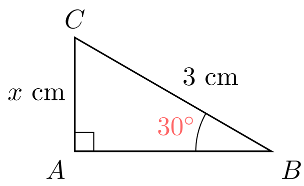
Answer: Relative to x:

- Adjacent side: $DE = 6$
- Hypotenuse: $EF = 10$

$$\begin{aligned}\cos(x) &= \frac{\text{ADJ}}{\text{HYP}} \\ &= \frac{6}{10} \\ &= \frac{3}{5}\end{aligned}$$

B.2 CALCULATING SIDE LENGTHS

Ex 13:



Calculate x.

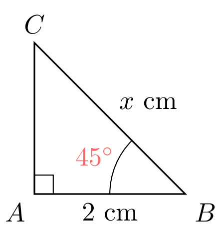
$$x \approx \boxed{1.50} \text{ cm (round to 2 decimal places)}$$

Answer: Relative to $\theta = 30°$:

- Opposite side: $AC = x$
- Hypotenuse: $BC = 3$

$$\begin{aligned}\sin(\theta) &= \frac{\text{OPP}}{\text{HYP}} \\ \sin(30°) &= \frac{x}{3} \\ x &= 3 \times \sin(30°) \\ x &= 3 \times 0.5 \\ x &= 1.50\,\text{cm}\end{aligned}$$

Ex 14:



Calculate x.

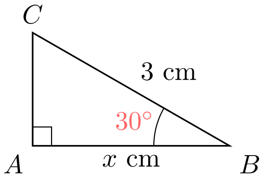
$$x \approx \boxed{2.83} \text{ cm (round to 2 decimal places)}$$

Answer: Relative to $\theta = 45°$:

- Adjacent side: $AB = 2$
- Hypotenuse: $BC = x$

$$\cos(\theta) = \frac{\text{ADJ}}{\text{HYP}}$$
$$\cos(45°) = \frac{2}{x}$$
$$x = \frac{2}{\cos(45°)}$$
$$x \approx 2.83\,\text{cm} \quad \text{(rounded to 2 decimal places)}$$

Ex 15:



Calculate x.

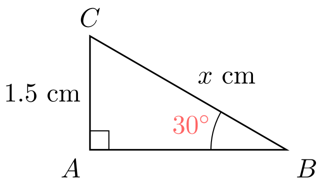
$x \approx$ 2.60 cm (round to 2 decimal places)

Answer: Relative to $\theta = 30°$:

- Adjacent side: $AB = x$
- Hypotenuse: $BC = 3$

$$\cos(\theta) = \frac{\text{ADJ}}{\text{HYP}}$$
$$\cos(30°) = \frac{x}{3}$$
$$x = 3 \times \cos(30°)$$
$$x \approx 2.60\,\text{cm} \quad \text{(rounded to 2 decimal places)}$$

Ex 16:



Calculate x.

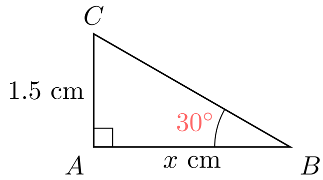
$x \approx$ 3.00 cm (round to 2 decimal places)

Answer: Relative to $\theta = 30°$:

- Opposite side: $AC = 1.5$
- Hypotenuse: $BC = x$

$$\sin(\theta) = \frac{\text{OPP}}{\text{HYP}}$$
$$\sin(30°) = \frac{1.5}{x}$$
$$x = \frac{1.5}{\sin(30°)}$$
$$x = \frac{1.5}{0.5}$$
$$x = 3.00\,\text{cm}$$

Ex 17:



Calculate x.

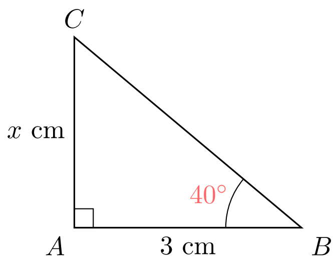
$x \approx$ 2.60 cm (round to 2 decimal places)

Answer: Relative to $\theta = 30°$:

- Opposite side: $AC = 1.5$
- Adjacent side: $AB = x$

$$\tan(\theta) = \frac{\text{OPP}}{\text{ADJ}}$$
$$\tan(30°) = \frac{1.5}{x}$$
$$x = \frac{1.5}{\tan(30°)}$$
$$x \approx 2.60\,\text{cm} \quad \text{(rounded to 2 decimal places)}$$

Ex 18:



Calculate x.

$x \approx$ 2.52 cm (round to 2 decimal places)

Answer: Relative to $\theta = 40°$:

- Opposite side: $AC = x$
- Adjacent side: $AB = 3$

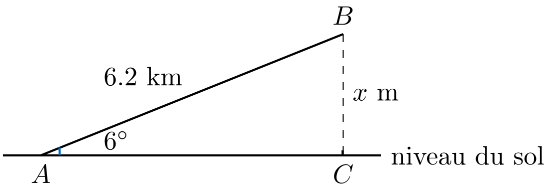
$$\tan(\theta) = \frac{\text{OPP}}{\text{ADJ}}$$
$$\tan(40°) = \frac{x}{3}$$
$$x = 3 \times \tan(40°)$$
$$x \approx 3 \times 0.8391$$
$$x \approx 2.52\,\text{cm} \quad \text{(rounded to 2 decimal places)}$$

B.3 SOLVING PROBLEMS

Ex 19: A cyclist in France rides up a long incline with an average rise of 6°. If he rides for 6.2 km, how far has he climbed vertically?

Students should calculate: $\sin(6°) = \frac{x}{6200}$, so $x = 6200 \times \sin(6°) \approx 648.62$ m.

Answer:



The cyclist rides 6.2 km (6200 m) up an incline with an average angle of 6°. This forms a right-angled triangle ABC with the right angle at C, where AB is the incline (hypotenuse) and BC is the vertical height. In $\triangle ABC$:

- Hypotenuse: $AB = 6200$ m (distance ridden).
- Opposite side (relative to $\angle A$): $BC = x$ (vertical height).
- Angle at A: 6°.

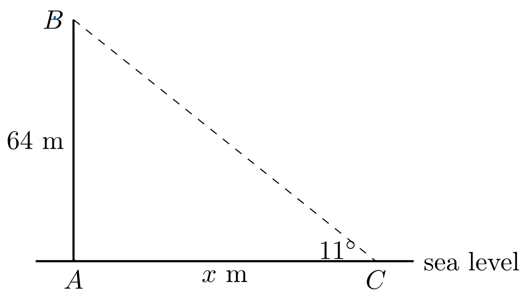
$$\begin{aligned}\sin(6°) &= \frac{\text{OPP}}{\text{HYP}} \\ &= \frac{x}{6200} \\ x &= 6200 \times \sin(6°) \\ &\approx 6200 \times 0.1045 \\ &\approx 648.62\,\text{m} \quad \text{(rounded to 2 decimal places)}\end{aligned}$$

Thus, the vertical height climbed is approximately 648.62 m.

Ex 20: The lamp in a lighthouse is 64 m above sea level. The angle of depression from the lamp to a fishing boat is 11°. How far horizontally is the boat from the lighthouse?

Students should calculate: $\tan(11°) = \frac{64}{x}$, so $x = \frac{64}{\tan(11°)} \approx$ 336.83 m.

Answer:



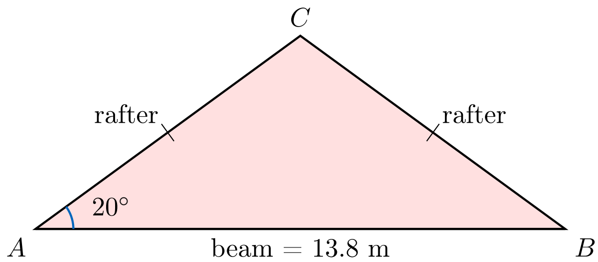
The lighthouse lamp (B) is 64 m above sea level (A), and the angle of depression from B to the fishing boat (C) is 11°. This forms a right-angled triangle ABC with the right angle at A. The angle of depression from B equals the angle of elevation at C (11°). In $\triangle ABC$:

- Opposite side (relative to $\angle C$): $AB = 64$ m (height of the lamp).
- Adjacent side: $AC = x$ (horizontal distance).
- Angle at C: 11°.

$$\begin{aligned}\tan(11°) &= \frac{\text{OPP}}{\text{ADJ}} \\ &= \frac{64}{x} \\ x &= \frac{64}{\tan(11°)} \\ &\approx \frac{64}{0.1944} \\ &\approx 336.83\,\text{m} \quad \text{(rounded to 2 decimal places)}\end{aligned}$$

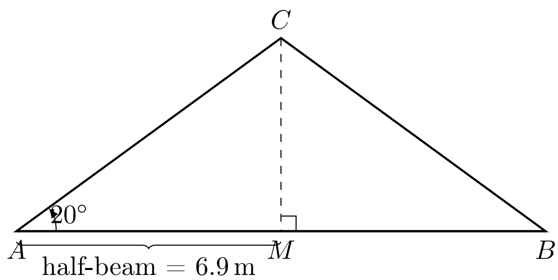
Thus, the horizontal distance from the boat to the lighthouse is approximately 336.83 m.

Ex 21: For the triangular roof truss illustrated, find the length of a rafter if the beam is 13.8 m and the pitch is 20°.



Students should write: $\cos(20°) = \frac{\text{half-beam}}{\text{rafter}} = \frac{6.9}{\text{rafter}}$, then calculate: rafter $= \frac{6.9}{\cos(20°)} \approx 7.35$ m.

Answer: Because the roof truss is isosceles, dropping the altitude from the ridge to the midpoint of the beam forms a right triangle whose hypotenuse is the rafter. Using the definition of the cosine in that right triangle:

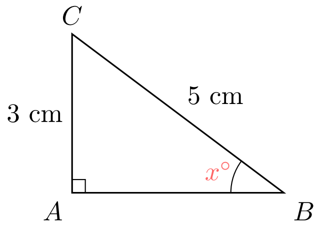


$$\begin{aligned}\cos(20°) &= \frac{\text{adjacent (half-beam)}}{\text{hypotenuse (rafter)}} \\ \text{rafter} &= \frac{\text{half-beam}}{\cos(20°)} \\ &= \frac{\frac{13.8}{2}}{\cos(20°)} \\ &= \frac{6.9}{\cos(20°)} \\ &\approx \frac{6.9}{0.9397} \\ &\approx 7.35\,\text{m} \quad \text{(rounded to 2 decimal places)}\end{aligned}$$

C INVERSE TRIGONOMETRIC FUNCTIONS

C.1 CALCULATING ANGLES

Ex 22:



Calculate the angle $x°$.

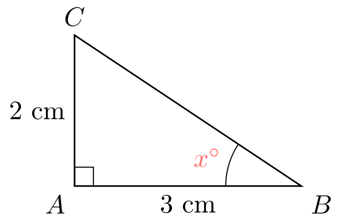
$$x° \approx \boxed{36.9}° \text{ (round to 1 decimal place)}$$

Answer: Relative to the angle x:

- Opposite side: $AC = 3$ cm
- Hypotenuse: $BC = 5$ cm

$$\begin{aligned} x° &= \sin^{-1}\left(\frac{\text{OPP}}{\text{HYP}}\right) \\ &= \sin^{-1}\left(\frac{3}{5}\right) \\ &= \sin^{-1}(0.6) \\ &\approx 36.9° \quad \text{(rounded to 1 decimal place)} \end{aligned}$$

Ex 23:



Calculate the angle $x°$.

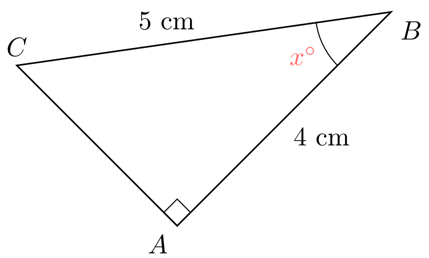
$$x° \approx \boxed{33.7}° \text{ (round to 1 decimal place)}$$

Answer: Relative to the angle x:

- Opposite side: $AC = 2$ cm
- Adjacent side: $AB = 3$ cm

$$\begin{aligned} x° &= \tan^{-1}\left(\frac{\text{OPP}}{\text{ADJ}}\right) \\ &= \tan^{-1}\left(\frac{2}{3}\right) \\ &= \tan^{-1}(0.6667) \\ &\approx 33.7° \quad \text{(rounded to 1 decimal place)} \end{aligned}$$

Ex 24:



Calculate the angle $x°$.

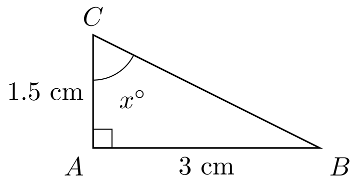
$$x° \approx \boxed{36.9}° \text{ (round to 1 decimal place)}$$

Answer: Relative to the angle x:

- Adjacent side: $AB = 4$ cm
- Hypotenuse: $BC = 5$ cm

$$\begin{aligned} x° &= \cos^{-1}\left(\frac{\text{ADJ}}{\text{HYP}}\right) \\ &= \cos^{-1}\left(\frac{4}{5}\right) \\ &= \cos^{-1}(0.8) \\ &\approx 36.9° \quad \text{(rounded to 1 decimal place)} \end{aligned}$$

Ex 25:



Calculate the angle $x°$.

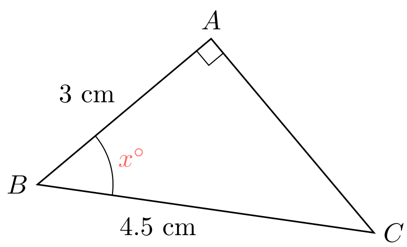
$$x° \approx \boxed{26.6}° \text{ (round to 1 decimal place)}$$

Answer: Relative to the angle x:

- Opposite side: $BC = 1.5$ cm
- Adjacent side: $AB = 3$ cm

$$\begin{aligned} x° &= \tan^{-1}\left(\frac{\text{OPP}}{\text{ADJ}}\right) \\ &= \tan^{-1}\left(\frac{1.5}{3}\right) \\ &= \tan^{-1}(0.5) \\ &\approx 26.6° \quad \text{(rounded to 1 decimal place)} \end{aligned}$$

Ex 26:



Calculate the angle $x°$.

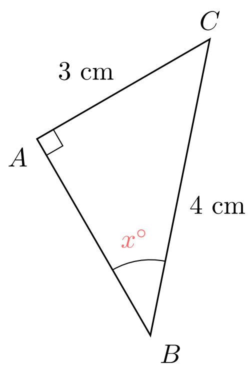
$$x° \approx \boxed{48.2}° \text{ (round to 1 decimal place)}$$

Answer: Relative to the angle x:

- Adjacent side: $AB = 3$ cm
- Hypotenuse: $BC = 4.5$ cm

$$
\begin{aligned}
x^\circ &= \cos^{-1}\left(\frac{\text{ADJ}}{\text{HYP}}\right)\\
&= \cos^{-1}\left(\frac{3}{4.5}\right)\\
&= \cos^{-1}\left(\frac{2}{3}\right)\\
&= \cos^{-1}(0.6667)\\
&\approx 48.2^\circ \quad \text{(rounded to 1 decimal place)}
\end{aligned}
$$

Ex 27:



Calculate the angle x°.

$x^\circ \approx \boxed{48.6}^\circ$ (round to 1 decimal place)

Answer: Relative to the angle x:

- Opposite side: $AC = 3$ cm
- Hypotenuse: $BC = 4$ cm

$$
\begin{aligned}
x^\circ &= \sin^{-1}\left(\frac{\text{OPP}}{\text{HYP}}\right)\\
&= \sin^{-1}\left(\frac{3}{4}\right)\\
&= \sin^{-1}(0.75)\\
&\approx 48.6^\circ \quad \text{(rounded to 1 decimal place)}
\end{aligned}
$$