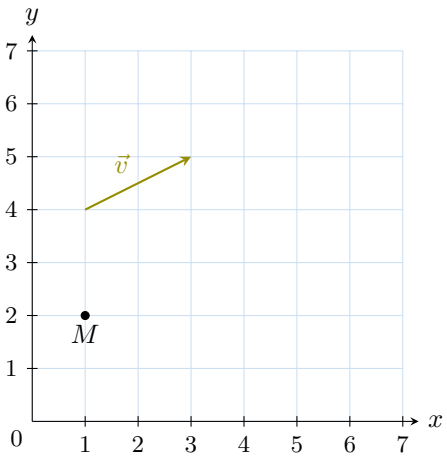
VECTORS

A DEFINITION

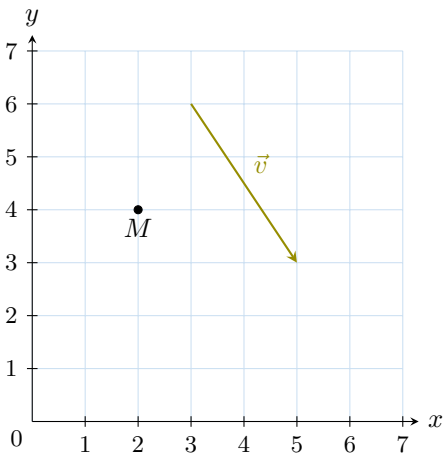
A.1 FINDING THE IMAGE OF A POINT

Ex 1: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



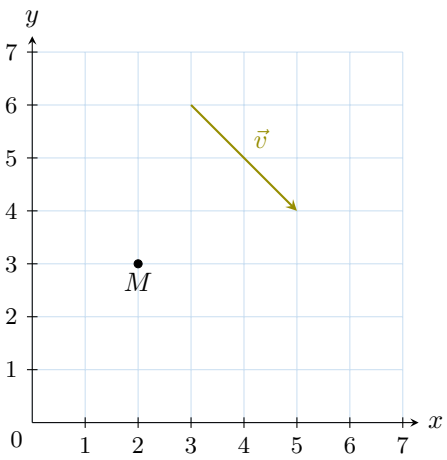
$M'($ ☐ ; ☐ $)$

Ex 2: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



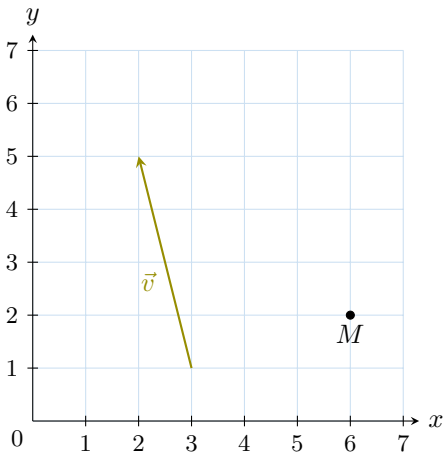
$M'($ ☐ ; ☐ $)$

Ex 3: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



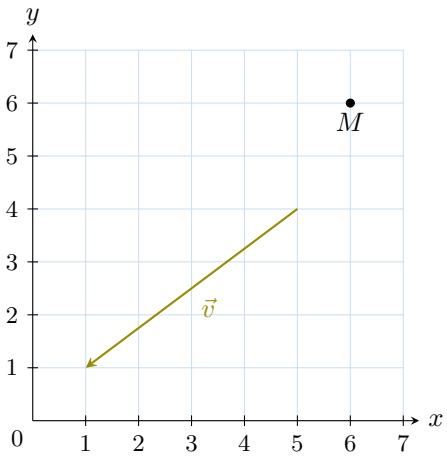
$M'($ ☐ ; ☐ $)$

Ex 4: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



$M'($ ☐ ; ☐ $)$

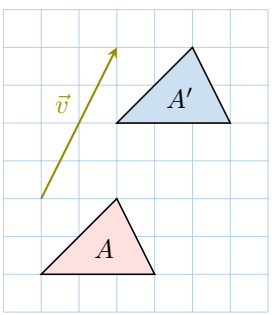
Ex 5: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



$M'($ ☐ ; ☐ $)$

A.2 TRANSLATION OF FIGURES

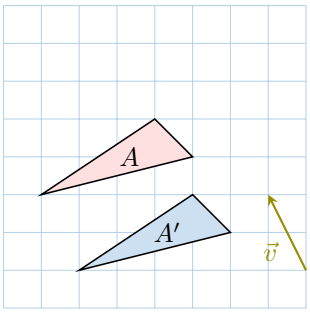
MCQ 6: Is the figure A' the image of figure A under a translation by vector $\vec{v}$?



☐ Yes

☐ No

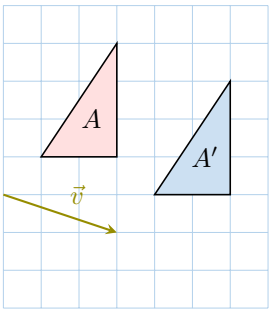
MCQ 7: Is the figure A' the image of figure A under a translation by vector $\vec{v}$?



☐ Yes

☐ No

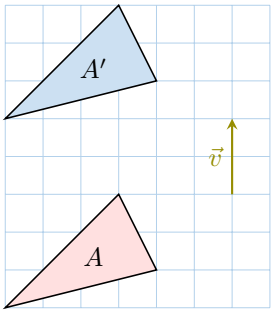
MCQ 8: Is the figure A' the image of figure A under a translation by vector $\vec{v}$?



☐ Yes

☐ No

MCQ 9: Is the figure A' the image of figure A under a translation by vector $\vec{v}$?

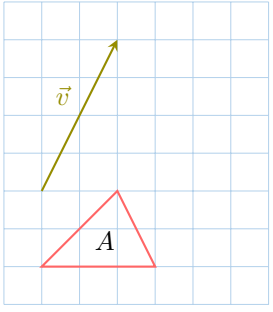


☐ Yes

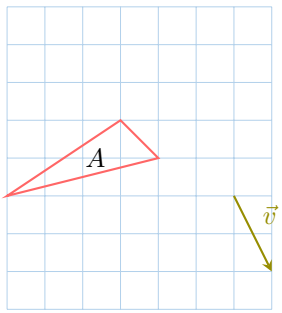
☐ No

A.3 DRAWING IMAGES FIGURES

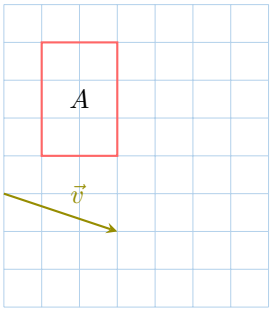
Ex 10: Draw the figure A', the image of figure A under a translation by vector $\vec{v}$.



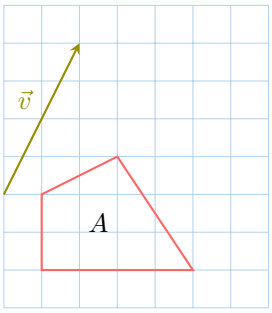
Ex 11: Draw the figure A', the image of figure A under a translation by vector $\vec{v}$.



Ex 12: Draw the figure A', the image of figure A under a translation by vector $\vec{v}$.

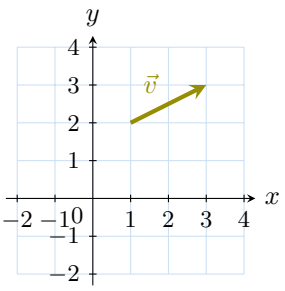


Ex 13: Draw the figure A', the image of figure A under a translation by vector $\vec{v}$.



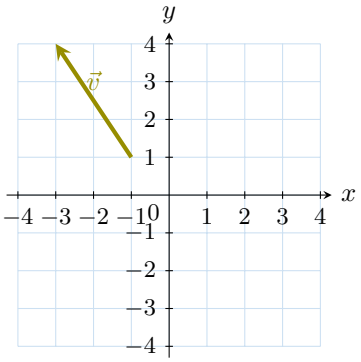
A.4 FINDING COMPONENTS OF A VECTOR

Ex 14: Find the components of the vector $\vec{v}$.



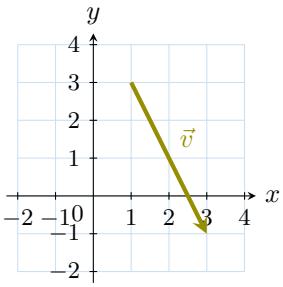
$\vec{v} =$ ( )

Ex 15: Find the components of the vector $\vec{v}$.



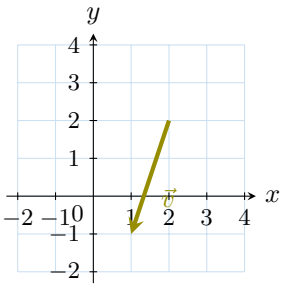
$\vec{v} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

Ex 16: Find the components of the vector $\vec{v}$.



$\vec{v} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

Ex 17: Find the components of the vector $\vec{v}$.



$\vec{v} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

A.5 REPRESENTING VECTORS ON A GRID

Ex 18: Draw the arrows diagram of $\vec{v}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

Ex 19: Draw the arrows diagram of $\vec{u} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

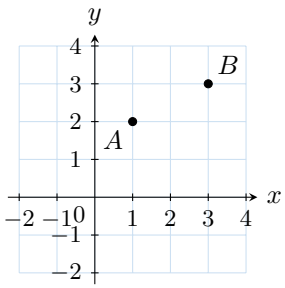
Ex 20: Draw the arrows diagram of $\vec{w} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$.

Ex 21: Draw the arrows diagram of $\vec{z} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$.

B TWO POINT NOTATION

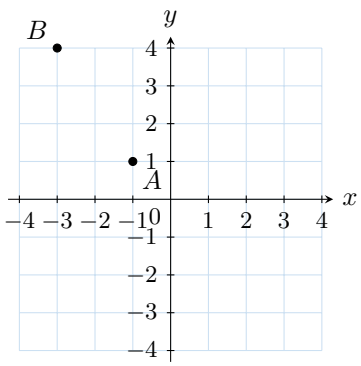
B.1 FINDING COMPONENTS OF A VECTOR

Ex 22: Find the components of the vector $\overrightarrow{AB}$.



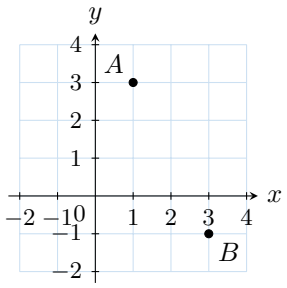
$\overrightarrow{AB} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

Ex 23: Find the components of the vector $\overrightarrow{AB}$.



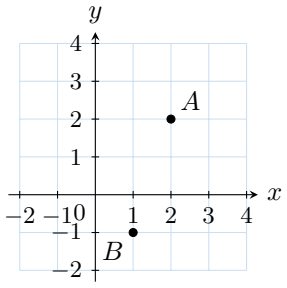
$\overrightarrow{AB} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

Ex 24: Find the components of the vector $\overrightarrow{AB}$.



$$\overrightarrow{AB} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

Ex 25: Find the components of the vector $\overrightarrow{AB}$.



$$\overrightarrow{AB} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

B.2 FINDING THE VECTOR COMPONENTS

Ex 26: For $A(1,2)$ and $B(3,3)$, find the components of the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

Ex 27: For $E(0,\,5)$ and $F(4,\,2)$, find the components of the vector $\overrightarrow{EF}$.

$$\overrightarrow{EF} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

Ex 28: For $B(-2,\,0)$ and $C(3,\,4)$, find the components of the vector $\overrightarrow{BC}$.

$$\overrightarrow{BC} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

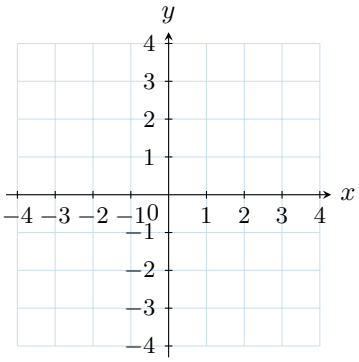
Ex 29: For $B(3,\,3)$ and $A(1,\,2)$, find the components of the vector $\overrightarrow{BA}$.

$$\overrightarrow{BA} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

B.3 PLACING A POINT USING A VECTOR

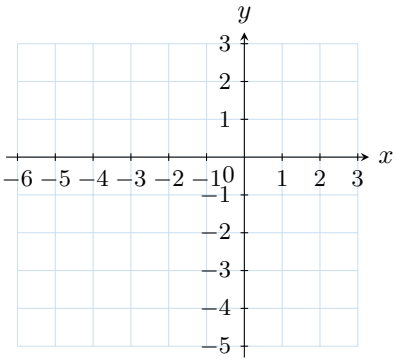
Ex 30:

1. Plot the point $A(-4;1)$.
2. Plot the point B such that $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.



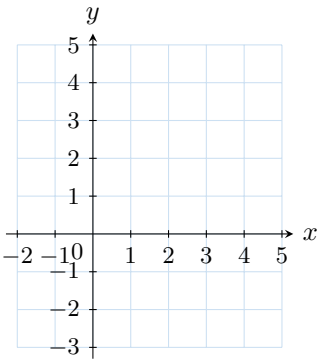
Ex 31:

1. Plot the point $C(1;\,-3)$.
2. Plot the point D such that $\overrightarrow{CD} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$.



Ex 32:

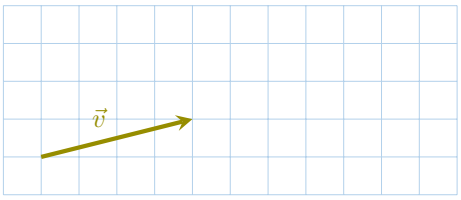
1. Plot the point $E(0;\,2)$.
2. Plot the point F such that $\overrightarrow{EF} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$.



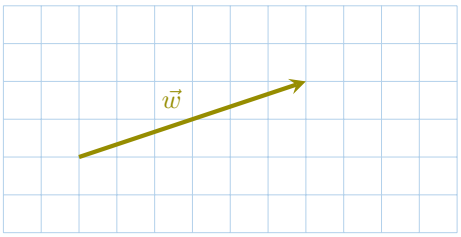
C EQUALITY BETWEEN VECTORS

C.1 DRAWING EQUAL VECTORS

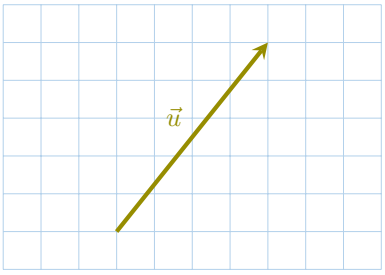
Ex 33: Draw a vector equal to $\vec{v}$.



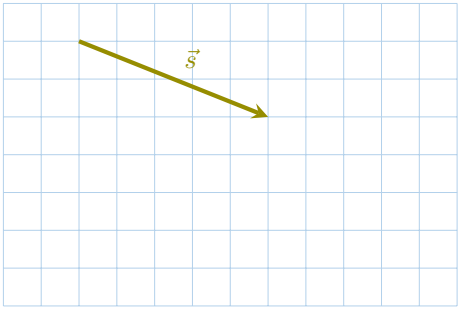
Ex 34: Draw a vector equal to $\vec{w}$.



Ex 35: Draw a vector equal to $\vec{u}$.



Ex 36: Draw a vector equal to $\vec{s}$.



C.2 FINDING THE COORDINATES OF A POINT WITH A GIVEN VECTOR

Ex 37: Let $A(2,\ 3)$, $B(5,\ 7)$, and $C(1,\ -2)$.
Find the coordinates of the point D such that $\overrightarrow{AB} = \overrightarrow{CD}$.

$$D = (\square, \square)$$

Ex 38: Let $A(0,\ 0)$, $B(4,\ 3)$, and $C(2,\ 1)$.
Find the coordinates of the point D such that $\overrightarrow{AB} = \overrightarrow{CD}$.

$$D = (\square, \square)$$

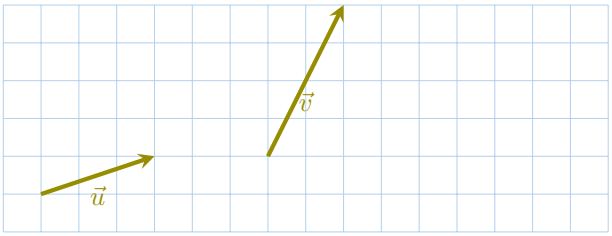
Ex 39: Let $A(-1,\ 2)$, $B(1,\ 5)$, and $C(3,\ -1)$.
Find the coordinates of the point D such that $\overrightarrow{AB} = \overrightarrow{CD}$.

$$D = (\square, \square)$$

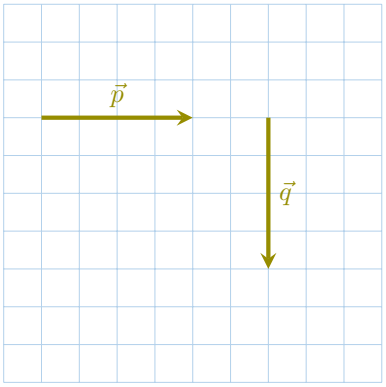
D ADDITION

D.1 DRAWING THE SUM OF TWO VECTORS

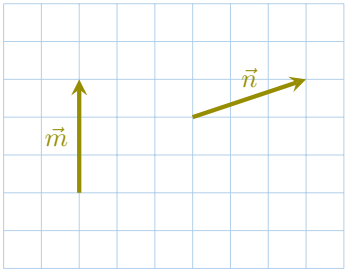
Ex 40: Draw the arrows diagram of $\vec{u} + \vec{v}$.



Ex 41: Draw the arrows diagram of $\vec{p} + \vec{q}$.



Ex 42: Draw the arrows diagram of $\vec{m} + \vec{n}$.



D.2 CALCULATING THE SUM OF VECTORS

Ex 43: Calculate the sum of the vectors $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$.

$$\vec{a} + \vec{b} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

Ex 44: Calculate the sum of the vectors $\vec{u} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$.

$$\vec{u} + \vec{v} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

Ex 45: Calculate the sum of the vectors $\vec{p} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$.

$$\vec{p}+\vec{q}=\begin{pmatrix}\square\\\square\end{pmatrix}$$

Ex 46: Calculate the sum of the vectors $\vec{m}=\begin{pmatrix}0\\-7\end{pmatrix}$ and $\vec{n}=\begin{pmatrix}5\\3\end{pmatrix}$.

$$\vec{m}+\vec{n}=\begin{pmatrix}\square\\\square\end{pmatrix}$$

D.3 RECOGNIZING SUMS OF VECTORS

MCQ 47: Calculate the sum of vectors: $\overrightarrow{AB}+\overrightarrow{BC}$.

- ☐ $\overrightarrow{CA}$
- ☐ $\overrightarrow{AC}$
- ☐ $\overrightarrow{BA}$
- ☐ $\overrightarrow{CB}$

MCQ 48: Calculate the sum of vectors: $\overrightarrow{BC}+\overrightarrow{AB}$.

- ☐ $\overrightarrow{CB}$
- ☐ $\overrightarrow{BA}$
- ☐ $\vec{0}$
- ☐ $\overrightarrow{AC}$

MCQ 49: Calculate the sum of vectors: $\overrightarrow{AB}+\overrightarrow{BA}$.

- ☐ $\overrightarrow{BA}$
- ☐ $\overrightarrow{AB}$
- ☐ $\vec{0}$

MCQ 50: Calculate the sum of vectors: $\overrightarrow{EA}+\overrightarrow{AB}+\overrightarrow{BC}$.

- ☐ $\overrightarrow{CE}$
- ☐ $\vec{0}$
- ☐ $\overrightarrow{AC}$
- ☐ $\overrightarrow{EC}$

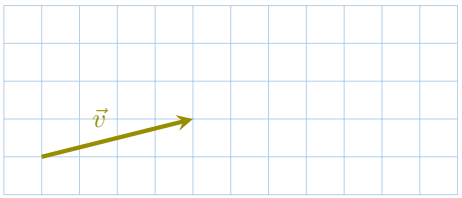
MCQ 51: Calculate the sum of vectors: $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}$.

- ☐ $\overrightarrow{AD}$
- ☐ $\overrightarrow{DA}$
- ☐ $\overrightarrow{BD}$
- ☐ $\vec{0}$

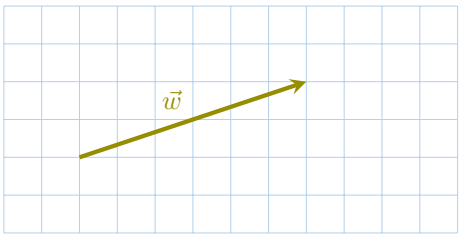
E SUBTRACTION

E.1 DRAWING THE NEGATIVE OF A VECTOR

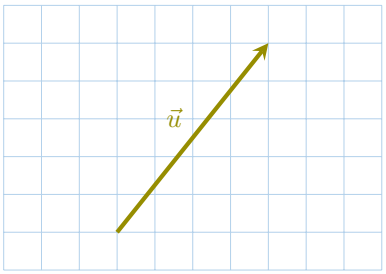
Ex 52: Draw the negative vector of $\vec{v}$.



Ex 53: Draw the negative vector of $\vec{w}$.



Ex 54: Draw the negative vector of $\vec{u}$.



E.2 CALCULATING THE NEGATIVE OF A VECTOR

Ex 55: Calculate the negative of the vector $\vec{a}=\begin{pmatrix}4\\-7\end{pmatrix}$.

$$-\vec{a}=\begin{pmatrix}\square\\\square\end{pmatrix}$$

Ex 56: Calculate the negative of the vector $\vec{b}=\begin{pmatrix}-3\\5\end{pmatrix}$.

$$-\vec{b}=\begin{pmatrix}\square\\\square\end{pmatrix}$$

Ex 57: Calculate the negative of the vector $\vec{u}=\begin{pmatrix}6\\2\end{pmatrix}$.

$$-\vec{u}=\begin{pmatrix}\square\\\square\end{pmatrix}$$

Ex 58: Calculate the negative of the vector $\vec{p}=\begin{pmatrix}0\\-8\end{pmatrix}$.

$$-\vec{p}=\begin{pmatrix}\square\\\square\end{pmatrix}$$

E.3 CALCULATING THE DIFFERENCE OF VECTORS

Ex 59: Calculate the difference of the vectors $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$.

$$\vec{a} - \vec{b} = \left(\square\right)$$

Ex 60: Calculate the difference of the vectors $\vec{u} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$.

$$\vec{u} - \vec{v} = \left(\square\right)$$

Ex 61: Calculate the difference of the vectors $\vec{p} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$.

$$\vec{p} - \vec{q} = \left(\square\right)$$

Ex 62: Calculate the difference of the vectors $\vec{m} = \begin{pmatrix} 0 \\ -7 \end{pmatrix}$ and $\vec{n} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$.

$$\vec{m} - \vec{n} = \left(\square\right)$$

F SCALAR MULTIPLICATION

F.1 MULTIPLYING A VECTOR BY A SCALAR

Ex 63: Calculate the product of the vector $\vec{b} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$ by 3.

$$3\vec{b} = \left(\square\right)$$

Ex 64: Calculate the product of the vector $\vec{u} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$ by -2.

$$-2\vec{u} = \left(\square\right)$$

Ex 65: Calculate the product of the vector $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ by -4.

$$-4\vec{a} = \left(\square\right)$$

Ex 66: Calculate the product of the vector $\vec{p} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$ by 0.5.

$$\frac{1}{2}\vec{p} = \left(\square\right)$$

F.2 CALCULATING LINEAR COMBINATIONS OF VECTORS

Ex 67: Calculate $3\vec{a} - \vec{b}$ where $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$.

$$3\vec{a} - \vec{b} = \left(\square\right)$$

Ex 68: Calculate $2\big(\vec{u} + 2\vec{v}\big)$ where $\vec{u} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

$$2\big(\vec{u} + 2\vec{v}\big) = \left(\square\right)$$

Ex 69: Calculate $4\vec{p} - 2\vec{q}$ where $\vec{p} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

$$4\vec{p} - 2\vec{q} = \left(\square\right)$$

Ex 70: Calculate $-3\vec{u} + 5\vec{v}$ where $\vec{u} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$.

$$-3\vec{u} + 5\vec{v} = \left(\square\right)$$

F.3 DETERMINING THE IMAGE OF A POINT UNDER A HOMOTHETY

Ex 71: Let $O(0,\ 0)$ and $M(3,\ -2)$. The point M' is the image of M by the homothety of center O and ratio $k = 2$ so that $2\,\overrightarrow{OM} = \overrightarrow{OM'}$.
Find the coordinates of M'.

$$M' = (\square, \square)$$

Ex 72: Let $A(2,\ -1)$ and $M(3,\ 1)$. The point M' is the image of M by the homothety of center A and ratio $k = -2$ so that $\overrightarrow{AM'} = -2\,\overrightarrow{AM}$.
Find the coordinates of M'.

$$M' = (\square, \square)$$

Ex 73: Let $A(2,\ -1)$ and $M(3,\ 1)$. The point M' is the image of M by the homothety of center A and ratio $k = 3$, so that $\overrightarrow{AM'} = 3\,\overrightarrow{AM}$.
Find the coordinates of M'.

$$M' = (\square, \square)$$

G MAGNITUDE OF A VECTOR

G.1 CALCULATING THE LENGTH OF A VECTOR

Ex 74: Calculate the length of $\vec{v} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$$\|\vec{v}\| = \square \text{ units}$$

Ex 75: Calculate the length of $\vec{p} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$

$\|\vec{p}\| =$ ☐ units

Ex 76: Calculate the length of $\vec{u} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

$\|\vec{u}\| =$ ☐ units

Ex 77: Calculate the length of $\vec{q} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\|\vec{q}\| =$ ☐ units

G.2 CALCULATING THE DISTANCE BETWEEN TWO POINTS

Ex 78: Let $A(2,\ 3)$ and $B(7,\ -1)$.

1. Calculate the vector $\overrightarrow{AB}$.

$\overrightarrow{AB} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

2. Calculate the distance AB.

$AB =$ ☐ units

Ex 79: Let $A(-2,\ 5)$ and $B(4,\ 2)$.

1. Calculate the vector $\overrightarrow{AB}$.

$\overrightarrow{AB} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

2. Calculate the distance AB.

$AB =$ ☐ units

Ex 80: Let $A(0,\ -2)$ and $B(-3,\ 6)$.

1. Calculate the vector $\overrightarrow{AB}$.

$\overrightarrow{AB} = \begin{pmatrix} \square \\ \square \end{pmatrix}$

2. Calculate the distance AB.

$AB =$ ☐ units

G.3 USING COORDINATES TO DETERMINE TRIANGLE TYPES

Ex 81: Let $A(0,\ 0)$, $B(6,\ 0)$, and $C(6,\ 8)$.

1. Calculate the lengths AB, BC, and CA.
   - $AB =$ ☐
   - $BC =$ ☐
   - $CA =$ ☐
2. Calculate the perimeter of triangle ABC.

Perimeter = ☐ units

Ex 82: Let $A(0,\ 0)$, $B(4,\ 0)$, and $C(2,\ 4)$.

1. Calculate the lengths AB, BC, and CA.
   - $AB =$ ☐
   - $BC =$ ☐
   - $CA =$ ☐
2. Is the triangle ABC isosceles?

☐ Yes
☐ No

Ex 83: Let $A(0,\ 0)$, $B(2,\ 2\sqrt{3})$, and $C(4,\ 0)$.

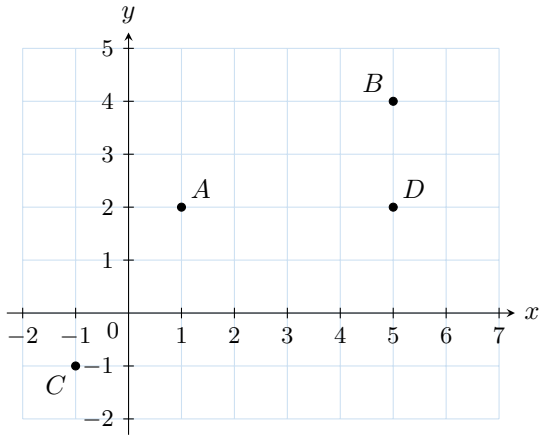
1. Calculate the lengths AB, BC, and CA.
   - $AB =$ ☐
   - $BC =$ ☐
   - $CA =$ ☐
2. Is the triangle ABC equilateral?

☐ Yes
☐ No

H COLINEARITY

H.1 TESTING PARALLELISM/ALIGNMENT USING VECTORS

Ex 84:



Let $A(1,\ 2)$, $B(5,\ 4)$, $C(-1,\ -1)$, and $D(5,\ 2)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

2. Calculate the vector $\overrightarrow{CD}$.

$$\overrightarrow{CD} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

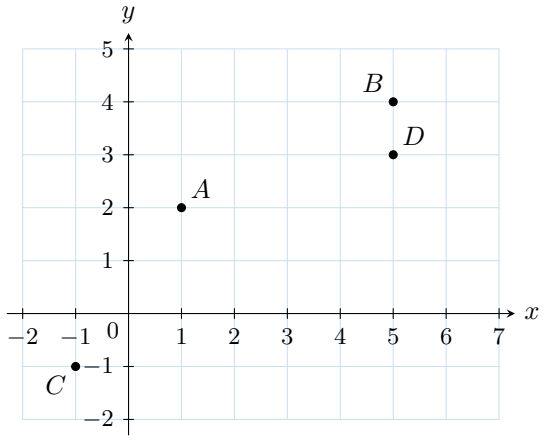
3. Calculate the determinant $\det(\overrightarrow{AB},\ \overrightarrow{CD})$.

$$\det(\overrightarrow{AB},\ \overrightarrow{CD}) = \square$$

4. Are the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ parallel?

☐ Yes
☐ No

Ex 85:



Let $A(1,\ 2)$, $B(5,\ 4)$, $C(-1,\ -1)$, and $D(5,\ 3)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

2. Calculate the vector $\overrightarrow{CD}$.

$$\overrightarrow{CD} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

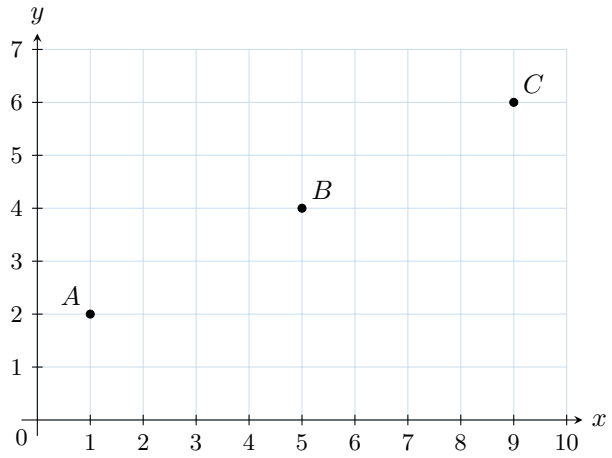
3. Calculate the determinant $\det(\overrightarrow{AB},\ \overrightarrow{CD})$.

$$\det(\overrightarrow{AB},\ \overrightarrow{CD}) = \square$$

4. Are the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ parallel?

☐ Yes
☐ No

Ex 86:



Let $A(1,\ 2)$, $B(5,\ 4)$, and $C(9,\ 6)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

2. Calculate the vector $\overrightarrow{AC}$.

$$\overrightarrow{AC} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

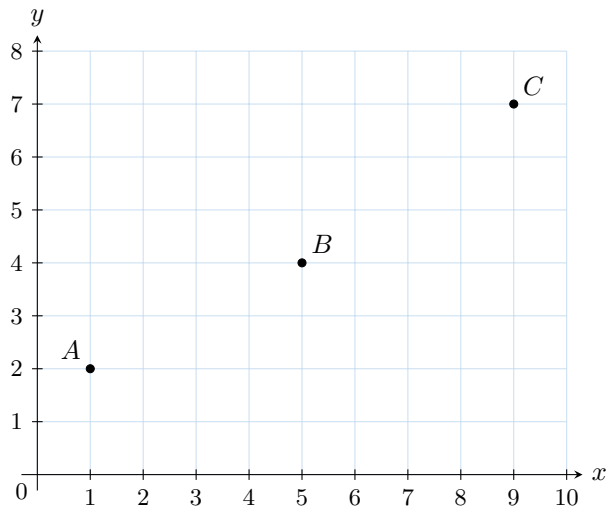
3. Calculate the determinant $\det(\overrightarrow{AB},\ \overrightarrow{AC})$.

$$\det(\overrightarrow{AB},\ \overrightarrow{AC}) = \square$$

4. Are the points A, B, and C aligned?

☐ Yes
☐ No

Ex 87:



Let $A(1,\ 2)$, $B(5,\ 4)$, and $C(9,\ 7)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

2. Calculate the vector $\overrightarrow{AC}$.

$$\overrightarrow{AC} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

3. Calculate the determinant $\det(\overrightarrow{AB},\ \overrightarrow{AC})$.

$$\det(\overrightarrow{AB},\ \overrightarrow{AC}) = \square$$

4. Are the points A, B, and C aligned?

☐ Yes
☐ No