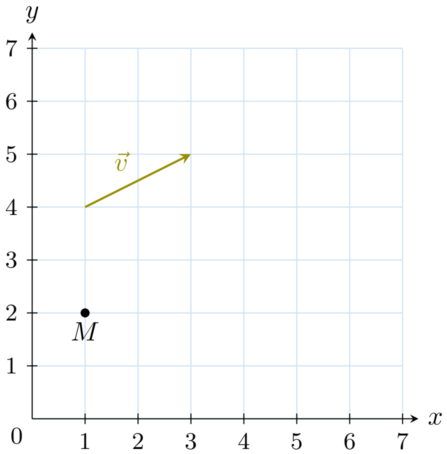
A DEFINITION

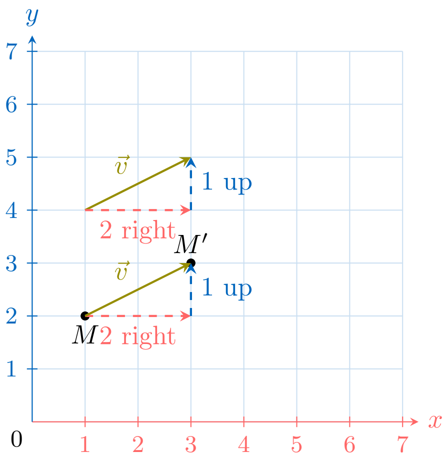
A.1 FINDING THE IMAGE OF A POINT

Ex 1: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



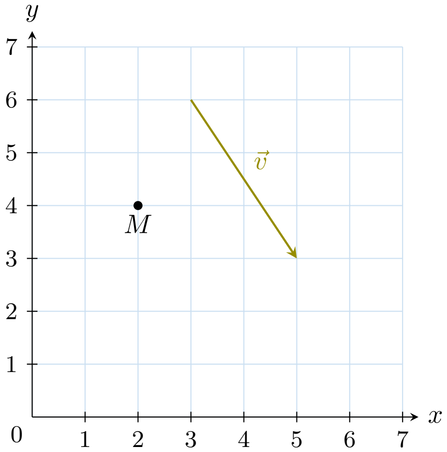
$M'(\boxed{3},\boxed{3})$

Answer:



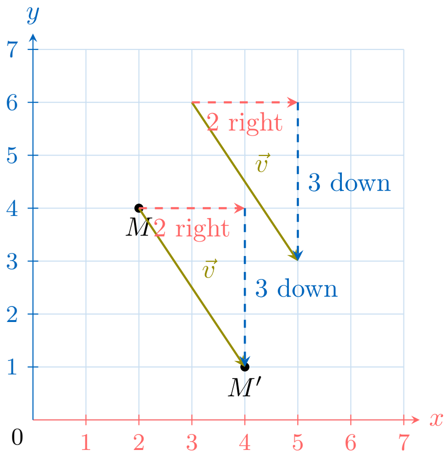
$M'(3,3)$

Ex 2: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



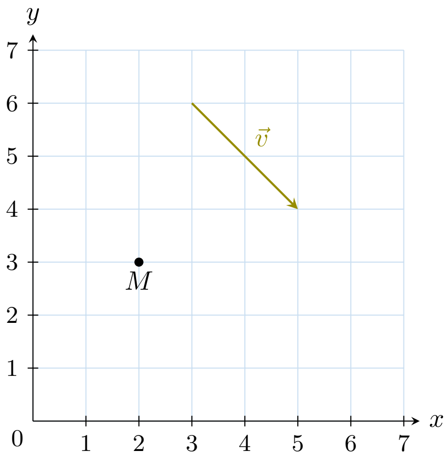
$M'(\boxed{4},\boxed{1})$

Answer:



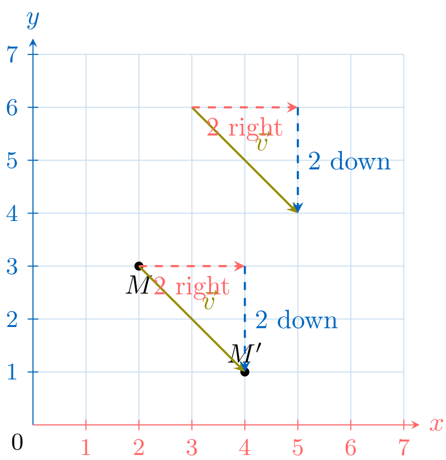
$M'(4,1)$

Ex 3: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



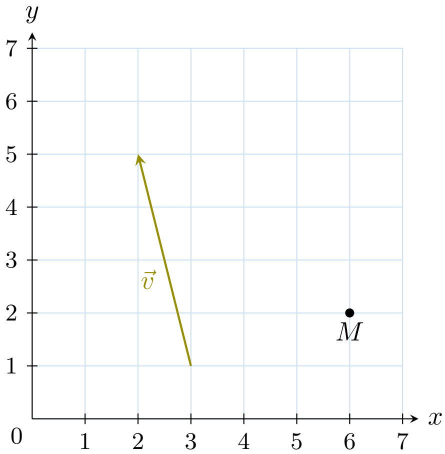
$M'(\boxed{4},\boxed{1})$

Answer:



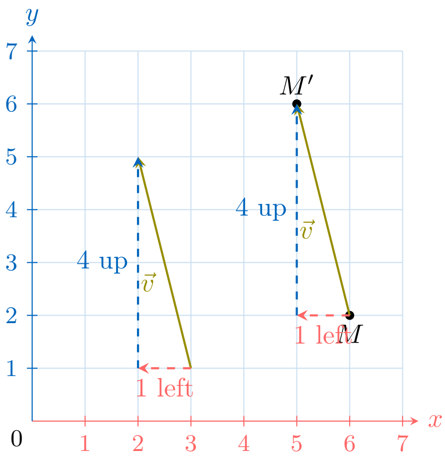
$M'(4,1)$

Ex 4: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



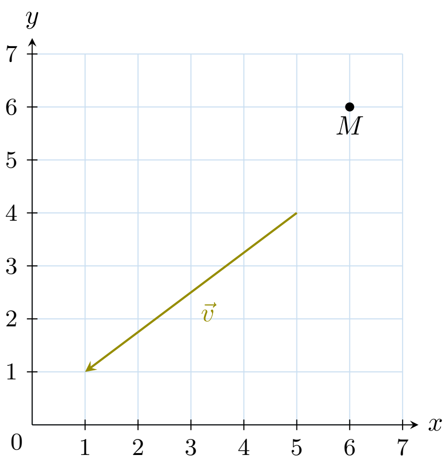
$M'(\boxed{5},\boxed{6})$

Answer:



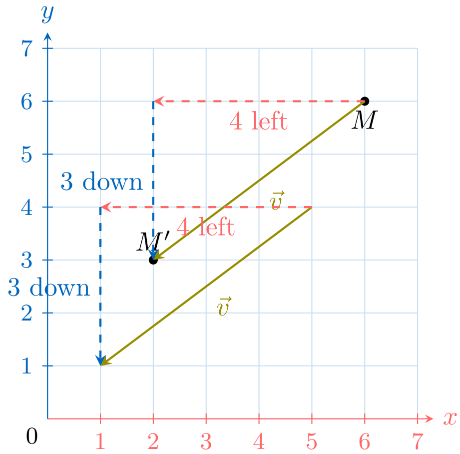
$M'(5,6)$

Ex 5: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



$M'(\boxed{2},\boxed{3})$

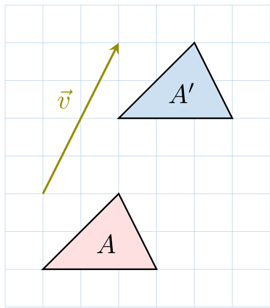
Answer:



$M'(2,3)$

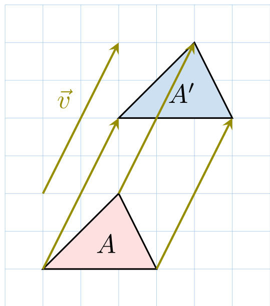
A.2 TRANSLATION OF FIGURES

MCQ 6: Is the figure A' the image of figure A under a translation by vector $\vec{v}$?

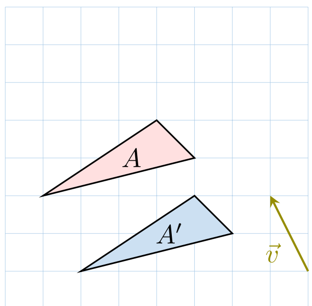


- [x] Yes
- [ ] No

Answer: Yes

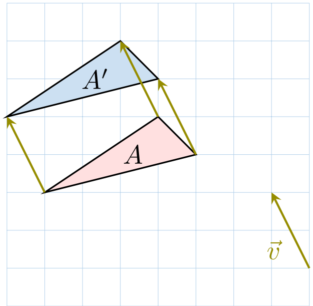


MCQ 7: Is the figure A' the image of figure A under a translation by vector $\vec{v}$?

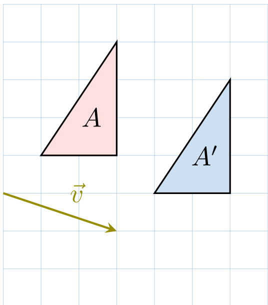


- [ ] Yes
- [x] No

Answer: No, the figure A' is misplaced. Here is where it should be.



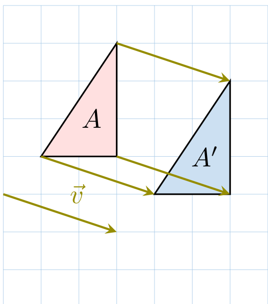
MCQ 8: Is the figure A' the image of figure A under a translation by vector $\vec{v}$?



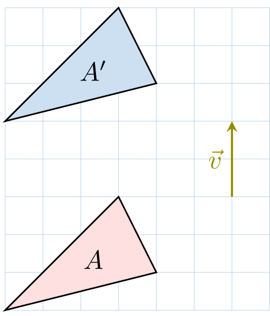
☒ Yes

☐ No

Answer: Yes



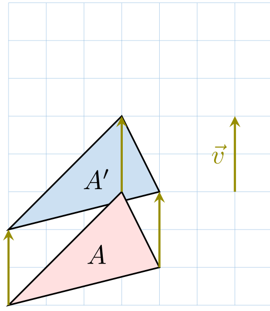
MCQ 9: Is the figure A' the image of figure A under a translation by vector $\vec{v}$?



☐ Yes

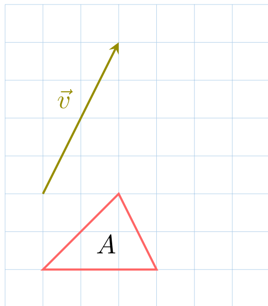
☒ No

Answer: No, the figure A' is misplaced. Here is where it should be.



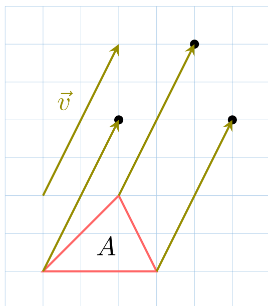
A.3 DRAWING IMAGES FIGURES

Ex 10: Draw the figure A', the image of figure A under a translation by vector $\vec{v}$.

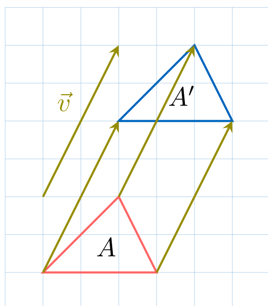


Answer:

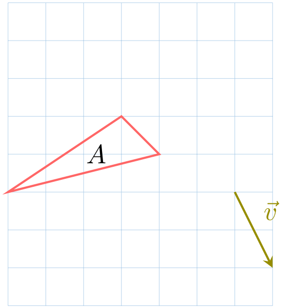
1. **Draw the image vertices**: For each vertex, translate it by the vector $\vec{v}$ by moving 2 units right and 4 units up from its original position. Place the new points on the grid.



2. **Draw the image figure**: Connect the image vertices with lines in the same order as the original figure.

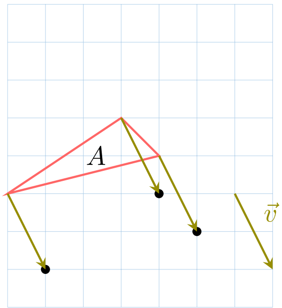


Ex 11: Draw the figure A', the image of figure A under a translation by vector $\vec{v}$.

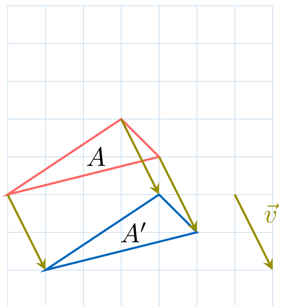


Answer:

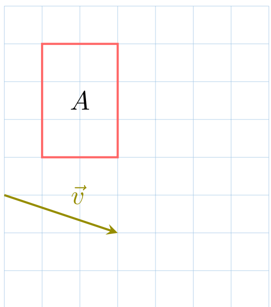
1. **Draw the image vertices**: For each vertex, translate it by the vector $\vec{v}$ by moving 1 unit right and 2 units down from its original position. Place the new points on the grid.



2. **Draw the image figure**: Connect the image vertices with lines in the same order as the original figure.

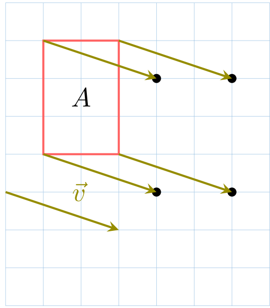


Ex 12: Draw the figure A', the image of figure A under a translation by vector $\vec{v}$.

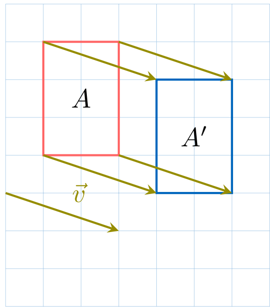


Answer:

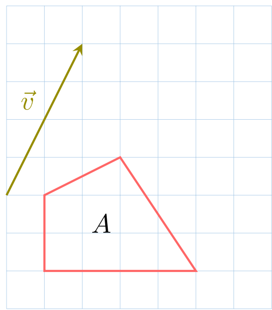
1. **Draw the image vertices**: For each vertex, translate it by the vector $\vec{v}$ by moving 3 units right and 1 unit down from its original position. Place the new points on the grid.



2. **Draw the image figure**: Connect the image vertices with lines in the same order as the original figure.

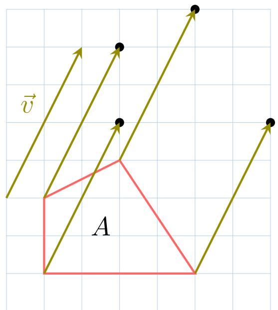


Ex 13: Draw the figure A', the image of figure A under a translation by vector $\vec{v}$.

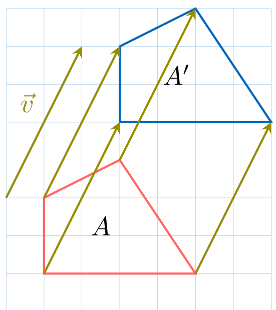


Answer:

1. **Draw the image vertices**: For each vertex, translate it by the vector $\vec{v}$ by moving 2 units right and 4 units up from its original position. Place the new points on the grid.

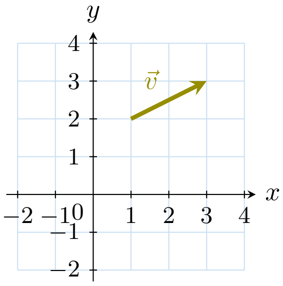


2. **Draw the image figure**: Connect the image vertices with lines in the same order as the original figure.



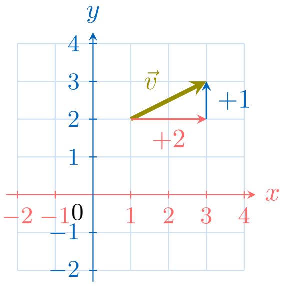
A.4 FINDING COMPONENTS OF A VECTOR

Ex 14: Find the components of the vector $\vec{v}$.



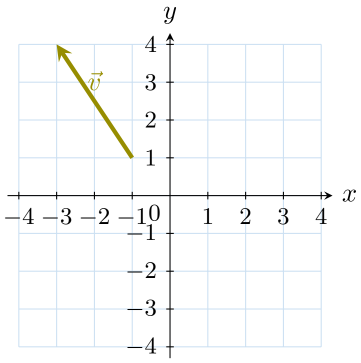
$$\vec{v} = \begin{pmatrix} \boxed{2} \\ \boxed{1} \end{pmatrix}$$

Answer:



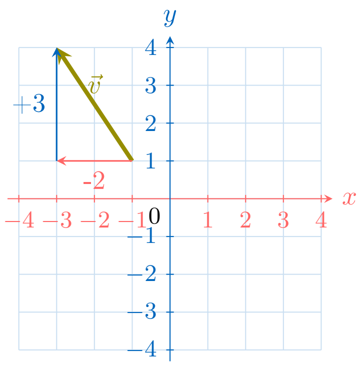
$$\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Ex 15: Find the components of the vector $\vec{v}$.



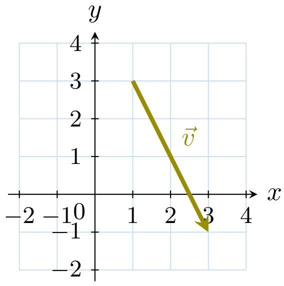
$$\vec{v} = \begin{pmatrix} \boxed{-2} \\ \boxed{3} \end{pmatrix}$$

Answer:



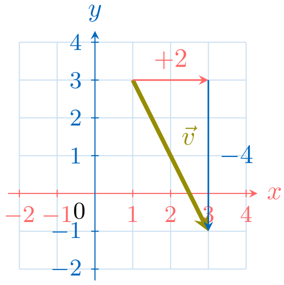
$$\vec{v} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

Ex 16: Find the components of the vector $\vec{v}$.



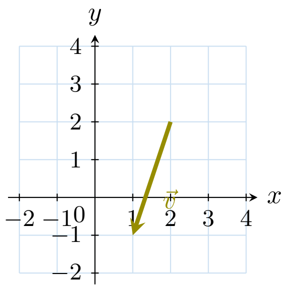
$$\vec{v} = \begin{pmatrix} \boxed{2} \\ \boxed{-4} \end{pmatrix}$$

Answer:



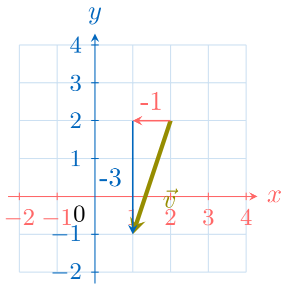
$$\vec{v} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

Ex 17: Find the components of the vector $\vec{v}$.



$$\vec{v} = \begin{pmatrix} \boxed{-1} \\ \boxed{-3} \end{pmatrix}$$

Answer:

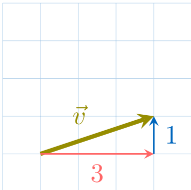


$$\vec{v} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$$

A.5 REPRESENTING VECTORS ON A GRID

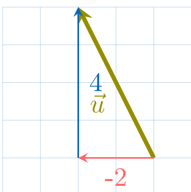
Ex 18: Draw the arrows diagram of $\vec{v}\begin{pmatrix}3\\1\end{pmatrix}$.

Answer:



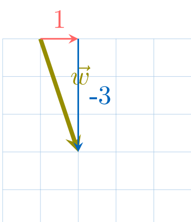
Ex 19: Draw the arrows diagram of $\vec{u}=\begin{pmatrix}-2\\4\end{pmatrix}$.

Answer:



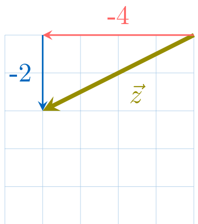
Ex 20: Draw the arrows diagram of $\vec{w}=\begin{pmatrix}1\\-3\end{pmatrix}$.

Answer:



Ex 21: Draw the arrows diagram of $\vec{z}=\begin{pmatrix}-4\\-2\end{pmatrix}$.

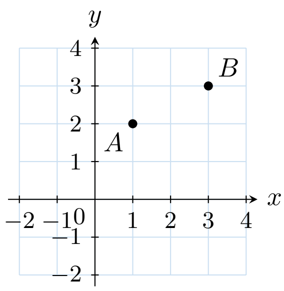
Answer:



B TWO POINT NOTATION

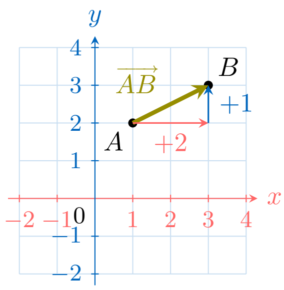
B.1 FINDING COMPONENTS OF A VECTOR

Ex 22: Find the components of the vector $\overrightarrow{AB}$.



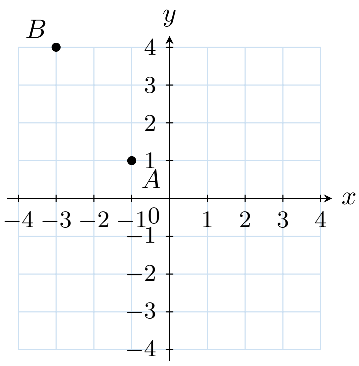
$$\overrightarrow{AB}=\begin{pmatrix}\boxed{2}\\\boxed{1}\end{pmatrix}$$

Answer:



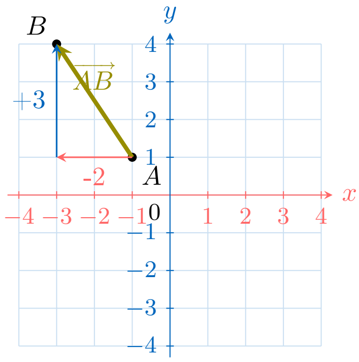
$$\overrightarrow{AB}=\begin{pmatrix}2\\1\end{pmatrix}$$

Ex 23: Find the components of the vector $\overrightarrow{AB}$.



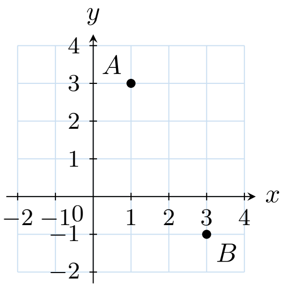
$$\overrightarrow{AB} = \begin{pmatrix} \boxed{-2} \\ \boxed{3} \end{pmatrix}$$

Answer:



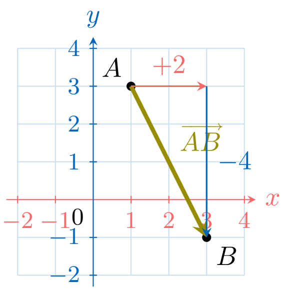
$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

Ex 24: Find the components of the vector $\overrightarrow{AB}$.



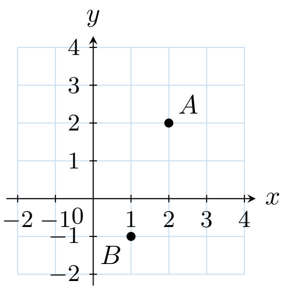
$$\overrightarrow{AB} = \begin{pmatrix} \boxed{2} \\ \boxed{-4} \end{pmatrix}$$

Answer:



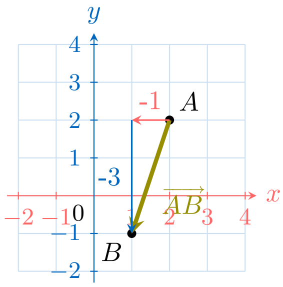
$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

Ex 25: Find the components of the vector $\overrightarrow{AB}$.



$$\overrightarrow{AB} = \begin{pmatrix} \boxed{-1} \\ \boxed{-3} \end{pmatrix}$$

Answer:



$$\overrightarrow{AB} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$$

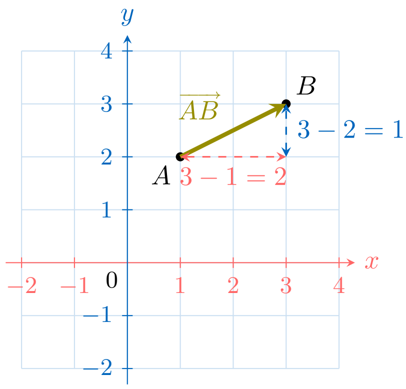
B.2 FINDING THE VECTOR COMPONENTS

Ex 26: For $A(1,2)$ and $B(3,3)$, find the components of the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{2} \\ \boxed{1} \end{pmatrix}$$

Answer:

$$\begin{aligned} \overrightarrow{AB} &= \begin{pmatrix} x_B - x_A \\ y_B - y_A \end{pmatrix} \\ &= \begin{pmatrix} 3-1 \\ 3-2 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ 1 \end{pmatrix} \end{aligned}$$

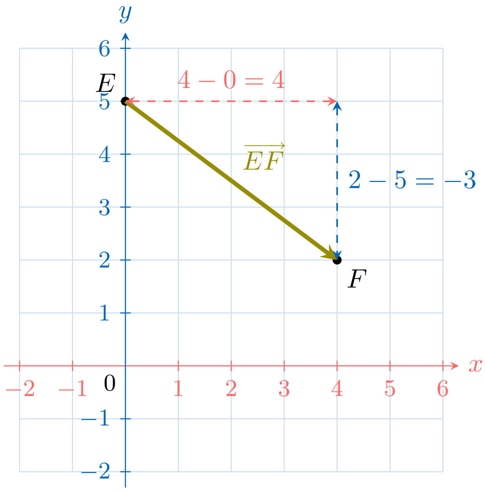


Ex 27: For $E(0, 5)$ and $F(4, 2)$, find the components of the vector $\overrightarrow{EF}$.

$$\overrightarrow{EF} = \begin{pmatrix} \boxed{4} \\ \boxed{-3} \end{pmatrix}$$

Answer:

$$\begin{aligned} \overrightarrow{EF} &= \begin{pmatrix} x_F - x_E \\ y_F - y_E \end{pmatrix} \\ &= \begin{pmatrix} 4-0 \\ 2-5 \end{pmatrix} \\ &= \begin{pmatrix} 4 \\ -3 \end{pmatrix} \end{aligned}$$

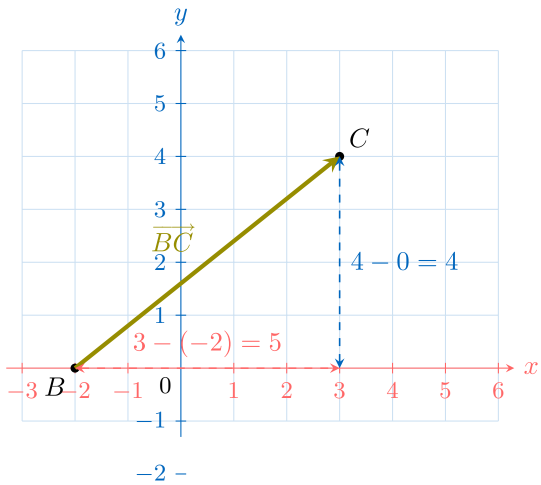


Ex 28: For $B(-2,\, 0)$ and $C(3,\, 4)$, find the components of the vector $\overrightarrow{BC}$.

$$\overrightarrow{BC} = \begin{pmatrix} \boxed{5} \\ \boxed{4} \end{pmatrix}$$

Answer:

$$\begin{aligned}\overrightarrow{BC} &= \begin{pmatrix} x_C - x_B \\ y_C - y_B \end{pmatrix} \\ &= \begin{pmatrix} 3-(-2) \\ 4-0 \end{pmatrix} \\ &= \begin{pmatrix} 5 \\ 4 \end{pmatrix}\end{aligned}$$

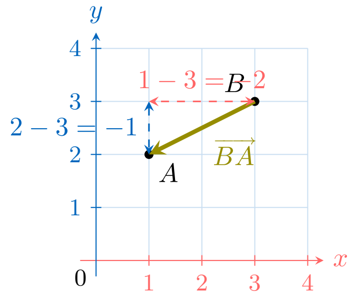


Ex 29: For $B(3,\, 3)$ and $A(1,\, 2)$, find the components of the vector $\overrightarrow{BA}$.

$$\overrightarrow{BA} = \begin{pmatrix} \boxed{-2} \\ \boxed{-1} \end{pmatrix}$$

Answer:

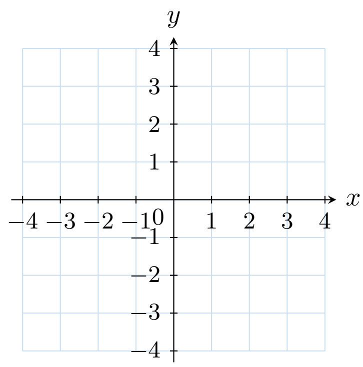
$$\begin{aligned}\overrightarrow{BA} &= \begin{pmatrix} x_A - x_B \\ y_A - y_B \end{pmatrix} \\ &= \begin{pmatrix} 1-3 \\ 2-3 \end{pmatrix} \\ &= \begin{pmatrix} -2 \\ -1 \end{pmatrix}\end{aligned}$$



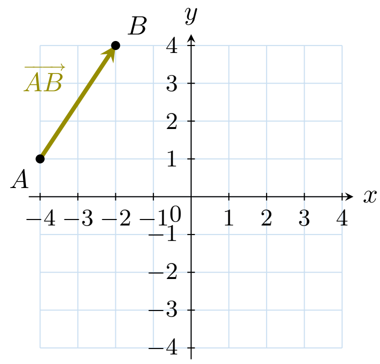
B.3 PLACING A POINT USING A VECTOR

Ex 30:

1. Plot the point $A(-4; 1)$.
2. Plot the point B such that $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.



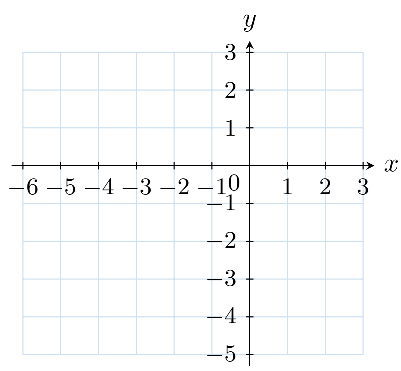
Answer:



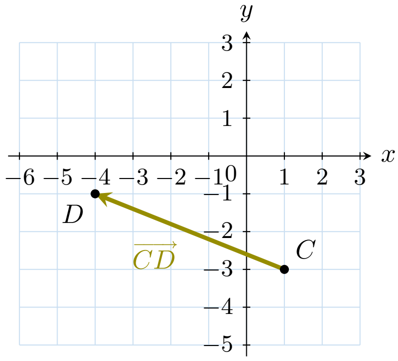
$A(-4; 1)$ and $B(-2; 4)$.

Ex 31:

1. Plot the point $C(1; -3)$.
2. Plot the point D such that $\overrightarrow{CD} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$.



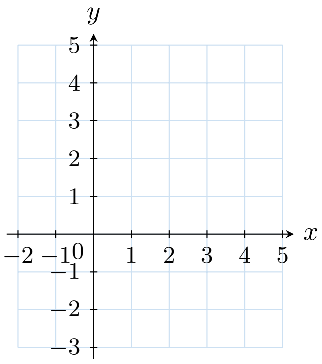
Answer:



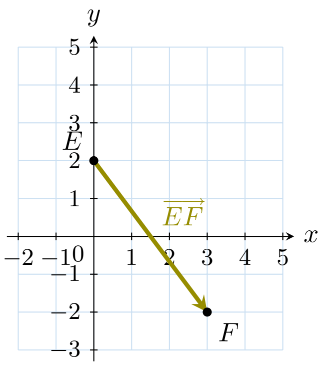
$C(1; -3)$ and $D(-4; -1)$.

Ex 32:

1. Plot the point $E(0; 2)$.
2. Plot the point F such that $\overrightarrow{EF} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$.



Answer:

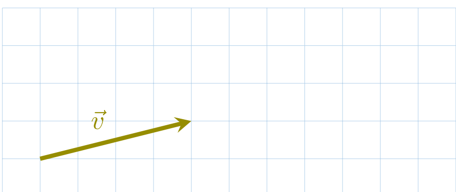


$E(0; 2)$ and $F(3; -2)$.

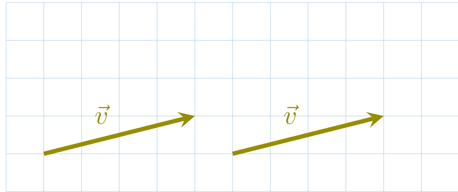
C EQUALITY BETWEEN VECTORS

C.1 DRAWING EQUAL VECTORS

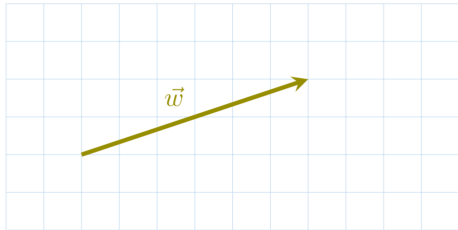
Ex 33: Draw a vector equal to $\vec{v}$.



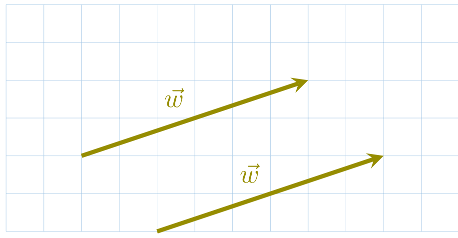
Answer: Draw a vector with the same direction, sense, and length as $\vec{v}$, starting from any point on the grid. For example:



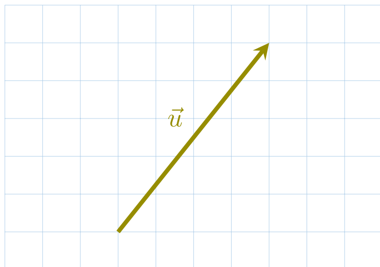
Ex 34: Draw a vector equal to $\vec{w}$.



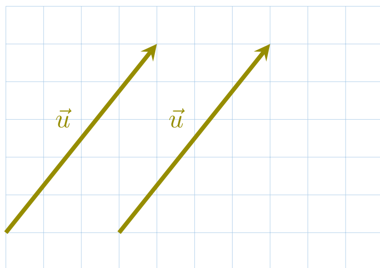
Answer: Draw a vector with the same direction, sense, and length as $\vec{w}$, starting from any point on the grid. For example:



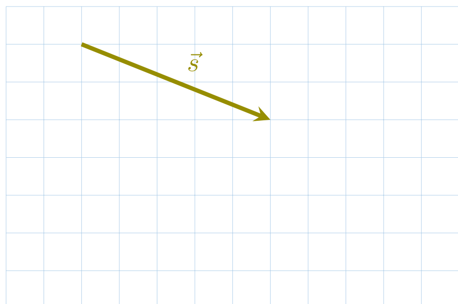
Ex 35: Draw a vector equal to $\vec{u}$.



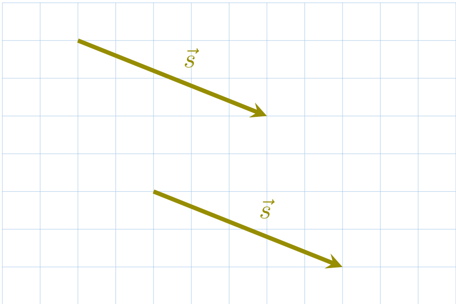
Answer: Draw a vector with the same direction, sense, and length as $\vec{u}$, starting from any point on the grid. For example:



Ex 36: Draw a vector equal to $\vec{s}$.



Answer: Draw a vector with the same direction, sense, and length as $\vec{s}$, starting from any point on the grid. For example:



C.2 FINDING THE COORDINATES OF A POINT WITH A GIVEN VECTOR

Ex 37: Let $A(2,\ 3)$, $B(5,\ 7)$, and $C(1,\ -2)$.
Find the coordinates of the point D such that $\overrightarrow{AB} = \overrightarrow{CD}$.

$$D = (\boxed{4}, \boxed{2})$$

Answer:

- First, compute the vectors:

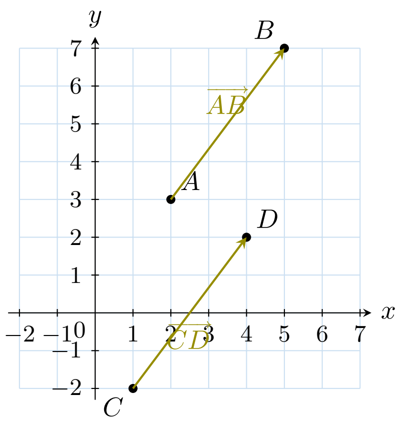
$$\overrightarrow{AB} = \begin{pmatrix} 5-2 \\ 7-3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\overrightarrow{CD} = \begin{pmatrix} x_D - 1 \\ y_D - (-2) \end{pmatrix} = \begin{pmatrix} x_D - 1 \\ y_D + 2 \end{pmatrix}$$

- Then, solve the equation:

$$\begin{aligned} \overrightarrow{AB} &= \overrightarrow{CD} \\ \begin{pmatrix} 3 \\ 4 \end{pmatrix} &= \begin{pmatrix} x_D - 1 \\ y_D + 2 \end{pmatrix} \\ 3 = x_D - 1 &\text{ and } 4 = y_D + 2 \\ x_D = 3 + 1 &\text{ et } y_D = 4 - 2 \\ x_D = 4 &\text{ and } y_D = +2 \end{aligned}$$

So, $D(4,\ 2)$.



Ex 38: Let $A(0,\ 0)$, $B(4,\ 3)$, and $C(2,\ 1)$.
Find the coordinates of the point D such that $\overrightarrow{AB} = \overrightarrow{CD}$.

$$D = (\boxed{6}, \boxed{4})$$

Answer:

- First, compute the vectors:

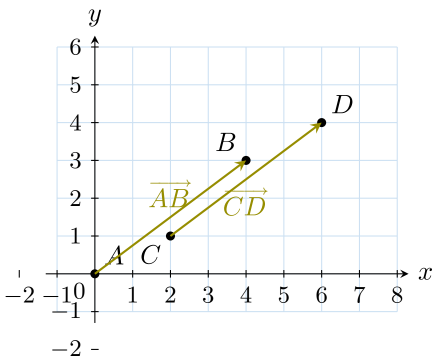
$$\overrightarrow{AB} = \begin{pmatrix} 4-0 \\ 3-0 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$\overrightarrow{CD} = \begin{pmatrix} x_D - 2 \\ y_D - 1 \end{pmatrix} = \begin{pmatrix} x_D - 2 \\ y_D - 1 \end{pmatrix}$$

- Then, solve the equation:

$$\begin{aligned} \overrightarrow{AB} &= \overrightarrow{CD} \\ \begin{pmatrix} 4 \\ 3 \end{pmatrix} &= \begin{pmatrix} x_D - 2 \\ y_D - 1 \end{pmatrix} \\ 4 = x_D - 2 &\text{ and } 3 = y_D - 1 \\ x_D = 6 &\text{ and } y_D = 4 \end{aligned}$$

So, $D(6,\ 4)$.



Ex 39: Let $A(-1,\ 2)$, $B(1,\ 5)$, and $C(3,\ -1)$.
Find the coordinates of the point D such that $\overrightarrow{AB} = \overrightarrow{CD}$.

$$D = (\boxed{5}, \boxed{2})$$

Answer:

- First, compute the vectors:

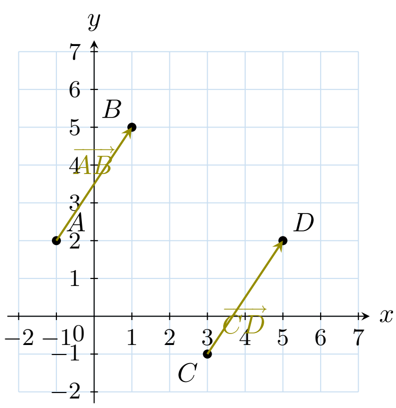
$$\overrightarrow{AB} = \begin{pmatrix} 1-(-1) \\ 5-2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\overrightarrow{CD} = \begin{pmatrix} x_D - 3 \\ y_D - (-1) \end{pmatrix} = \begin{pmatrix} x_D - 3 \\ y_D + 1 \end{pmatrix}$$

- Then, solve the equation:

$$\begin{aligned} \overrightarrow{AB} &= \overrightarrow{CD} \\ \begin{pmatrix} 2 \\ 3 \end{pmatrix} &= \begin{pmatrix} x_D - 3 \\ y_D + 1 \end{pmatrix} \\ 2 = x_D - 3 &\text{ and } 3 = y_D + 1 \\ x_D = 5 &\text{ and } y_D = 2 \end{aligned}$$

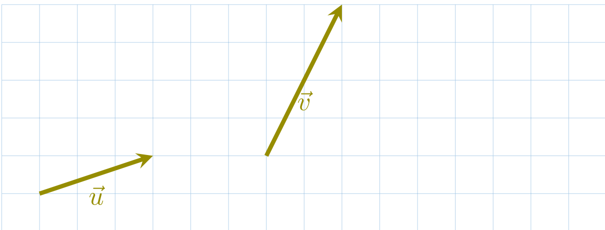
So, $D(5,\ 2)$.



D ADDITION

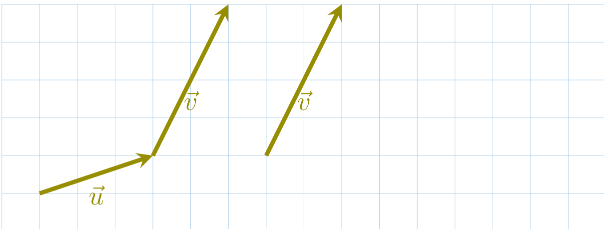
D.1 DRAWING THE SUM OF TWO VECTORS

Ex 40: Draw the arrows diagram of $\vec{u}+\vec{v}$.

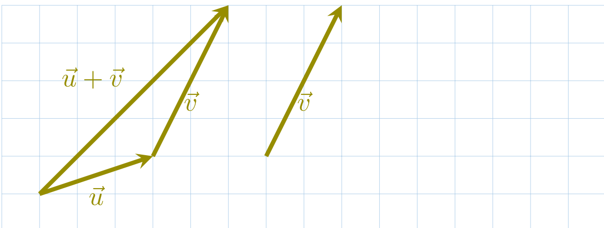


Answer: To add $\vec{u}$ and $\vec{v}$:

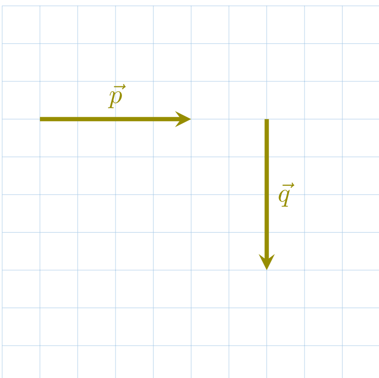
1. At the arrowhead end of $\vec{u}$, draw $\vec{v}$ starting from there (keep the same length and direction).



2. Draw the resulting vector from the start of $\vec{u}$ to the tip of the new $\vec{v}$. This vector is $\vec{u}+\vec{v}$.

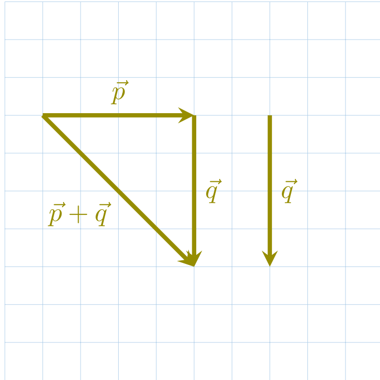


Ex 41: Draw the arrows diagram of $\vec{p}+\vec{q}$.

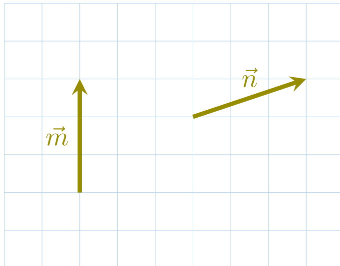


Answer: To add $\vec{p}$ and $\vec{q}$:

1. Place $\vec{q}$ starting at the tip of $\vec{p}$ (preserving its direction and length).
2. Draw the vector from the tail of $\vec{p}$ to the tip of this new $\vec{q}$. This is $\vec{p}+\vec{q}$.

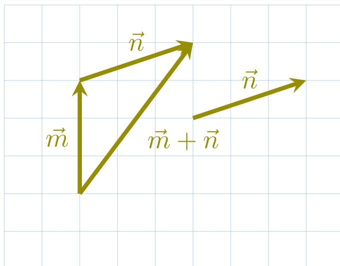


Ex 42: Draw the arrows diagram of $\vec{m}+\vec{n}$.



Answer: To add $\vec{m}$ and $\vec{n}$:

1. Draw $\vec{n}$ starting at the tip of $\vec{m}$ (same direction and length as the original).
2. Draw the resulting vector from the origin of $\vec{m}$ to the tip of this new $\vec{n}$. This is $\vec{m}+\vec{n}$.



D.2 CALCULATING THE SUM OF VECTORS

Ex 43: Calculate the sum of the vectors $\vec{a}=\begin{pmatrix}2\\-3\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}-5\\4\end{pmatrix}$.

$$\vec{a}+\vec{b}=\begin{pmatrix}\boxed{-3}\\\boxed{1}\end{pmatrix}$$

Answer:

$$\begin{aligned}\vec{a}+\vec{b}&=\begin{pmatrix}2\\-3\end{pmatrix}+\begin{pmatrix}-5\\4\end{pmatrix}\\&=\begin{pmatrix}2+(-5)\\(-3)+4\end{pmatrix}\\&=\begin{pmatrix}-3\\1\end{pmatrix}\end{aligned}$$

Ex 44: Calculate the sum of the vectors $\vec{u}=\begin{pmatrix}4\\2\end{pmatrix}$ and $\vec{v}=\begin{pmatrix}-1\\5\end{pmatrix}$.

$$\vec{u}+\vec{v}=\begin{pmatrix}\boxed{3}\\\boxed{7}\end{pmatrix}$$

Answer:

$$\begin{aligned}\vec{u}+\vec{v}&=\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}-1\\5\end{pmatrix}\\&=\begin{pmatrix}4+(-1)\\2+5\end{pmatrix}\\&=\begin{pmatrix}3\\7\end{pmatrix}\end{aligned}$$

Ex 45: Calculate the sum of the vectors $\vec{p}=\begin{pmatrix}-3\\6\end{pmatrix}$ and $\vec{q}=\begin{pmatrix}8\\-4\end{pmatrix}$.

$$\vec{p}+\vec{q}=\begin{pmatrix}\boxed{5}\\\boxed{2}\end{pmatrix}$$

Answer:

$$\begin{aligned}\vec{p}+\vec{q}&=\begin{pmatrix}-3\\6\end{pmatrix}+\begin{pmatrix}8\\-4\end{pmatrix}\\&=\begin{pmatrix}(-3)+8\\6+(-4)\end{pmatrix}\\&=\begin{pmatrix}5\\2\end{pmatrix}\end{aligned}$$

Ex 46: Calculate the sum of the vectors $\vec{m}=\begin{pmatrix}0\\-7\end{pmatrix}$ and $\vec{n}=\begin{pmatrix}5\\3\end{pmatrix}$.

$$\vec{m}+\vec{n}=\begin{pmatrix}\boxed{5}\\\boxed{-4}\end{pmatrix}$$

Answer:

$$\begin{aligned}\vec{m}+\vec{n}&=\begin{pmatrix}0\\-7\end{pmatrix}+\begin{pmatrix}5\\3\end{pmatrix}\\&=\begin{pmatrix}0+5\\(-7)+3\end{pmatrix}\\&=\begin{pmatrix}5\\-4\end{pmatrix}\end{aligned}$$

D.3 RECOGNIZING SUMS OF VECTORS

MCQ 47: Calculate the sum of vectors: $\overrightarrow{AB}+\overrightarrow{BC}$.

- ☐ $\overrightarrow{CA}$
- ☒ $\overrightarrow{AC}$
- ☐ $\overrightarrow{BA}$
- ☐ $\overrightarrow{CB}$

Answer:

$$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}\quad\text{(by Chasles' relation)}$$

MCQ 48: Calculate the sum of vectors: $\overrightarrow{BC}+\overrightarrow{AB}$.

- ☐ $\overrightarrow{CB}$
- ☐ $\overrightarrow{BA}$
- ☐ $\vec{0}$
- ☒ $\overrightarrow{AC}$

Answer:

$$\begin{aligned}\overrightarrow{BC}+\overrightarrow{AB}&=\overrightarrow{AB}+\overrightarrow{BC}\\&=\overrightarrow{AC}\quad(\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}\ \text{b}\end{aligned}$$

MCQ 49: Calculate the sum of vectors: $\overrightarrow{AB}+\overrightarrow{BA}$.

- ☐ $\overrightarrow{BA}$
- ☐ $\overrightarrow{AB}$
- ☒ $\vec{0}$

Answer:

$$\begin{aligned}\overrightarrow{AB}+\overrightarrow{BA}&=\overrightarrow{AA}\\&=\vec{0}\end{aligned}$$

MCQ 50: Calculate the sum of vectors: $\overrightarrow{EA}+\overrightarrow{AB}+\overrightarrow{BC}$.

- ☐ $\overrightarrow{CE}$
- ☐ $\vec{0}$
- ☐ $\overrightarrow{AC}$
- ☒ $\overrightarrow{EC}$

Answer:

$$\begin{aligned}\overrightarrow{EA}+\overrightarrow{AB}+\overrightarrow{BC}&=\overrightarrow{EB}+\overrightarrow{AC}\\&=\overrightarrow{EC}\end{aligned}$$

MCQ 51: Calculate the sum of vectors: $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}$.

- ☒ $\overrightarrow{AD}$
- ☐ $\overrightarrow{DA}$
- ☐ $\overrightarrow{BD}$
- ☐ $\vec{0}$

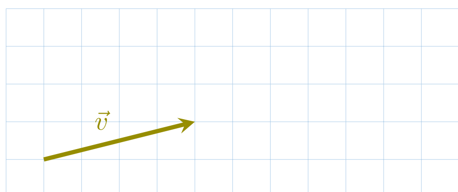
Answer:

$$\begin{aligned}\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}&=\overrightarrow{AC}+\overrightarrow{CD}\\&=\overrightarrow{AD}\end{aligned}$$

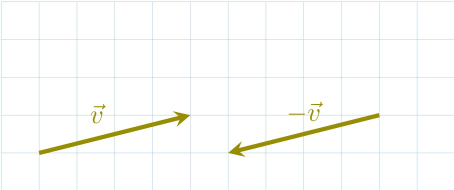
E SUBTRACTION

E.1 DRAWING THE NEGATIVE OF A VECTOR

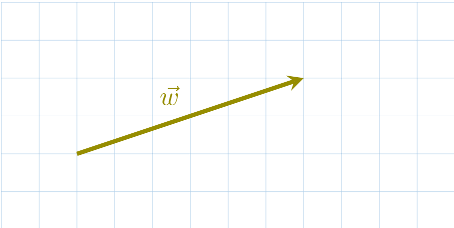
Ex 52: Draw the negative vector of $\vec{v}$.



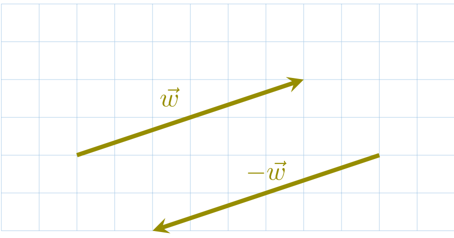
Answer: Draw a vector with the same direction, **opposite** sense, and length as $\vec{v}$, starting from any point on the grid. For example:



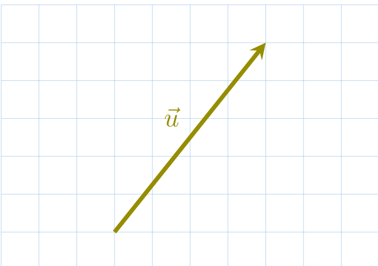
Ex 53: Draw the negative vector of $\vec{w}$.



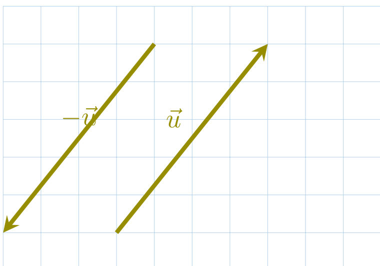
Answer: Draw a vector with the same direction, **opposite** sense, and length as $\vec{w}$, starting from any point on the grid. For example:



Ex 54: Draw the negative vector of $\vec{u}$.



Answer: Draw a vector with the same direction, **opposite** sense, and length as $\vec{u}$, starting from any point on the grid. For example:



E.2 CALCULATING THE NEGATIVE OF A VECTOR

Ex 55: Calculate the negative of the vector $\vec{a} = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$.

$$-\vec{a} = \begin{pmatrix} \boxed{-4} \\ \boxed{7} \end{pmatrix}$$

Answer:

$$\begin{aligned} -\vec{a} &= -\begin{pmatrix} 4 \\ -7 \end{pmatrix} \\ &= \begin{pmatrix} -4 \\ 7 \end{pmatrix} \end{aligned}$$

Ex 56: Calculate the negative of the vector $\vec{b} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$.

$$-\vec{b} = \begin{pmatrix} \boxed{3} \\ \boxed{-5} \end{pmatrix}$$

Answer:

$$\begin{aligned} -\vec{b} &= -\begin{pmatrix} -3 \\ 5 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ -5 \end{pmatrix} \end{aligned}$$

Ex 57: Calculate the negative of the vector $\vec{u} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$.

$$-\vec{u} = \begin{pmatrix} \boxed{-6} \\ \boxed{-2} \end{pmatrix}$$

Answer:

$$\begin{aligned} -\vec{u} &= -\begin{pmatrix} 6 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} -6 \\ -2 \end{pmatrix} \end{aligned}$$

Ex 58: Calculate the negative of the vector $\vec{p} = \begin{pmatrix} 0 \\ -8 \end{pmatrix}$.

$$-\vec{p} = \begin{pmatrix} \boxed{0} \\ \boxed{8} \end{pmatrix}$$

Answer:

$$\begin{aligned} -\vec{p} &= -\begin{pmatrix} 0 \\ -8 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 8 \end{pmatrix} \end{aligned}$$

E.3 CALCULATING THE DIFFERENCE OF VECTORS

Ex 59: Calculate the difference of the vectors $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$.

$$\vec{a} - \vec{b} = \begin{pmatrix} \boxed{7} \\ \boxed{-7} \end{pmatrix}$$

Answer:

$$\begin{aligned} \vec{a} - \vec{b} &= \begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} -5 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} 2 - (-5) \\ -3 - 4 \end{pmatrix} \\ &= \begin{pmatrix} 7 \\ -7 \end{pmatrix} \end{aligned}$$

Ex 60: Calculate the difference of the vectors $\vec{u} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$.

$$\vec{u}-\vec{v}=\begin{pmatrix}\boxed{5}\\ \boxed{-3}\end{pmatrix}$$

Answer:

$$\begin{aligned}\vec{u}-\vec{v}&=\begin{pmatrix}4\\2\end{pmatrix}-\begin{pmatrix}-1\\5\end{pmatrix}\\&=\begin{pmatrix}4-(-1)\\2-5\end{pmatrix}\\&=\begin{pmatrix}5\\-3\end{pmatrix}\end{aligned}$$

Ex 61: Calculate the difference of the vectors $\vec{p}=\begin{pmatrix}-3\\6\end{pmatrix}$ and $\vec{q}=\begin{pmatrix}8\\-4\end{pmatrix}$.

$$\vec{p}-\vec{q}=\begin{pmatrix}\boxed{-11}\\ \boxed{10}\end{pmatrix}$$

Answer:

$$\begin{aligned}\vec{p}-\vec{q}&=\begin{pmatrix}-3\\6\end{pmatrix}-\begin{pmatrix}8\\-4\end{pmatrix}\\&=\begin{pmatrix}-3-8\\6-(-4)\end{pmatrix}\\&=\begin{pmatrix}-11\\10\end{pmatrix}\end{aligned}$$

Ex 62: Calculate the difference of the vectors $\vec{m}=\begin{pmatrix}0\\-7\end{pmatrix}$ and $\vec{n}=\begin{pmatrix}5\\3\end{pmatrix}$.

$$\vec{m}-\vec{n}=\begin{pmatrix}\boxed{-5}\\ \boxed{-10}\end{pmatrix}$$

Answer:

$$\begin{aligned}\vec{m}-\vec{n}&=\begin{pmatrix}0\\-7\end{pmatrix}-\begin{pmatrix}5\\3\end{pmatrix}\\&=\begin{pmatrix}0-5\\-7-3\end{pmatrix}\\&=\begin{pmatrix}-5\\-10\end{pmatrix}\end{aligned}$$

F SCALAR MULTIPLICATION

F.1 MULTIPLYING A VECTOR BY A SCALAR

Ex 63: Calculate the product of the vector $\vec{b}=\begin{pmatrix}-5\\4\end{pmatrix}$ by 3.

$$3\vec{b}=\begin{pmatrix}\boxed{-15}\\ \boxed{12}\end{pmatrix}$$

Answer:

$$\begin{aligned}3\vec{b}&=3\times\begin{pmatrix}-5\\4\end{pmatrix}\\&=\begin{pmatrix}3\times(-5)\\3\times 4\end{pmatrix}\\&=\begin{pmatrix}-15\\12\end{pmatrix}\end{aligned}$$

Ex 64: Calculate the product of the vector $\vec{u}=\begin{pmatrix}0\\6\end{pmatrix}$ by -2.

$$-2\vec{u}=\begin{pmatrix}\boxed{0}\\ \boxed{-12}\end{pmatrix}$$

Answer:

$$\begin{aligned}-2\vec{u}&=-2\times\begin{pmatrix}0\\6\end{pmatrix}\\&=\begin{pmatrix}-2\times 0\\-2\times 6\end{pmatrix}\\&=\begin{pmatrix}0\\-12\end{pmatrix}\end{aligned}$$

Ex 65: Calculate the product of the vector $\vec{a}=\begin{pmatrix}2\\-3\end{pmatrix}$ by -4.

$$-4\vec{a}=\begin{pmatrix}\boxed{-8}\\ \boxed{12}\end{pmatrix}$$

Answer:

$$\begin{aligned}-4\vec{a}&=-4\times\begin{pmatrix}2\\-3\end{pmatrix}\\&=\begin{pmatrix}-4\times 2\\-4\times(-3)\end{pmatrix}\\&=\begin{pmatrix}-8\\12\end{pmatrix}\end{aligned}$$

Ex 66: Calculate the product of the vector $\vec{p}=\begin{pmatrix}7\\-1\end{pmatrix}$ by 0.5.

$$\frac{1}{2}\vec{p}=\begin{pmatrix}\boxed{3.5}\\ \boxed{-0.5}\end{pmatrix}$$

Answer:

$$\begin{aligned}0.5\vec{p}&=0.5\times\begin{pmatrix}7\\-1\end{pmatrix}\\&=\begin{pmatrix}0.5\times 7\\0.5\times(-1)\end{pmatrix}\\&=\begin{pmatrix}3.5\\-0.5\end{pmatrix}\end{aligned}$$

F.2 CALCULATING LINEAR COMBINATIONS OF VECTORS

Ex 67: Calculate $3\vec{a}-\vec{b}$ where $\vec{a}=\begin{pmatrix}2\\-3\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}-5\\4\end{pmatrix}$.

$$3\vec{a}-\vec{b}=\begin{pmatrix}\boxed{11}\\ \boxed{-13}\end{pmatrix}$$

Answer:

$$\begin{aligned}3\vec{a}-\vec{b}&=3\times\begin{pmatrix}2\\-3\end{pmatrix}-\begin{pmatrix}-5\\4\end{pmatrix}\\&=\begin{pmatrix}3\times 2\\3\times(-3)\end{pmatrix}-\begin{pmatrix}-5\\4\end{pmatrix}\\&=\begin{pmatrix}6\\-9\end{pmatrix}-\begin{pmatrix}-5\\4\end{pmatrix}\\&=\begin{pmatrix}6-(-5)\\-9-4\end{pmatrix}\\&=\begin{pmatrix}6+(+5)\\-9-4\end{pmatrix}\\&=\begin{pmatrix}11\\-13\end{pmatrix}\end{aligned}$$

Ex 68: Calculate $2\left(\vec{u}+2\vec{v}\right)$ where $\vec{u}=\begin{pmatrix}1\\-2\end{pmatrix}$ and $\vec{v}=\begin{pmatrix}3\\5\end{pmatrix}$.

$$2\left(\vec{u}+2\vec{v}\right)=\begin{pmatrix}\boxed{14}\\\boxed{22}\end{pmatrix}$$

Answer:

$$\begin{aligned}2\left(\vec{u}+2\vec{v}\right)&=2\left(\begin{pmatrix}1\\-2\end{pmatrix}+2\times\begin{pmatrix}3\\5\end{pmatrix}\right)\\&=2\left(\begin{pmatrix}1\\-2\end{pmatrix}+\begin{pmatrix}6\\10\end{pmatrix}\right)\\&=2\begin{pmatrix}1+6\\-2+10\end{pmatrix}\\&=2\begin{pmatrix}7\\8\end{pmatrix}\\&=\begin{pmatrix}14\\16\end{pmatrix}\end{aligned}$$

Ex 69: Calculate $4\vec{p}-2\vec{q}$ where $\vec{p}=\begin{pmatrix}-1\\3\end{pmatrix}$ and $\vec{q}=\begin{pmatrix}2\\-5\end{pmatrix}$.

$$4\vec{p}-2\vec{q}=\begin{pmatrix}\boxed{-8}\\\boxed{22}\end{pmatrix}$$

Answer:

$$\begin{aligned}4\vec{p}-2\vec{q}&=4\begin{pmatrix}-1\\3\end{pmatrix}-2\begin{pmatrix}2\\-5\end{pmatrix}\\&=\begin{pmatrix}4\times-1\\4\times3\end{pmatrix}-\begin{pmatrix}2\times2\\2\times-5\end{pmatrix}\\&=\begin{pmatrix}-4\\12\end{pmatrix}-\begin{pmatrix}4\\-10\end{pmatrix}\\&=\begin{pmatrix}-4-4\\12-(-10)\end{pmatrix}\\&=\begin{pmatrix}-8\\22\end{pmatrix}\end{aligned}$$

Ex 70: Calculate $-3\vec{u}+5\vec{v}$ where $\vec{u}=\begin{pmatrix}2\\0\end{pmatrix}$ and $\vec{v}=\begin{pmatrix}-1\\4\end{pmatrix}$.

$$-3\vec{u}+5\vec{v}=\begin{pmatrix}\boxed{-11}\\\boxed{20}\end{pmatrix}$$

Answer:

$$\begin{aligned}-3\vec{u}+5\vec{v}&=-3\begin{pmatrix}2\\0\end{pmatrix}+5\begin{pmatrix}-1\\4\end{pmatrix}\\&=\begin{pmatrix}-6\\0\end{pmatrix}+\begin{pmatrix}-5\\20\end{pmatrix}\\&=\begin{pmatrix}-6+(-5)\\0+20\end{pmatrix}\\&=\begin{pmatrix}-11\\20\end{pmatrix}\end{aligned}$$

F.3 DETERMINING THE IMAGE OF A POINT UNDER A HOMOTHETY

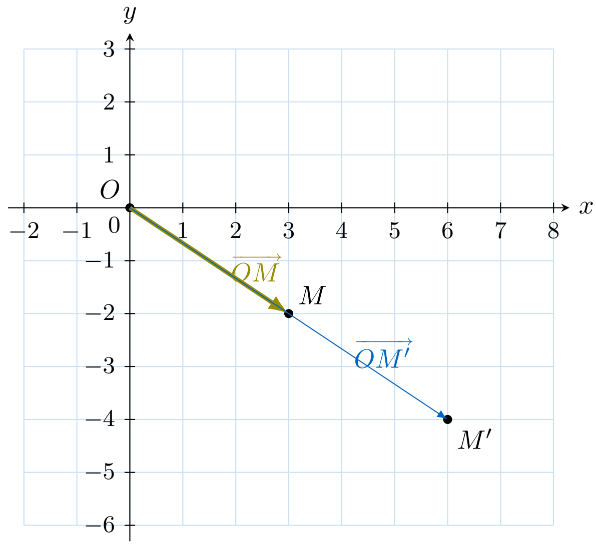
Ex 71: Let $O(0,\ 0)$ and $M(3,\ -2)$. The point M' is the image of M by the homothety of center O and ratio $k=2$ so that $2\,\overrightarrow{OM}=\overrightarrow{OM'}$.
Find the coordinates of M'.

$$M'=(\boxed{6},\ \boxed{-4})$$

Answer:

- $$\begin{aligned}\overrightarrow{OM'}&=2\ \overrightarrow{OM}\\&=2\begin{pmatrix}3\\-2\end{pmatrix}\\&=\begin{pmatrix}6\\-4\end{pmatrix}\end{aligned}$$
- $$\overrightarrow{OM'}=\begin{pmatrix}x_{M'}-x_O\\y_{M'}-y_O\end{pmatrix}$$
  $$\begin{pmatrix}6\\-4\end{pmatrix}=\begin{pmatrix}x_{M'}-0\\y_{M'}-0\end{pmatrix}$$
  So $x_{M'}-0=6$ given $x_{M'}=6$ and $y_{M'}-0=-4$ given $y_{M'}=-4$

So $M'(6,\ -4)$.



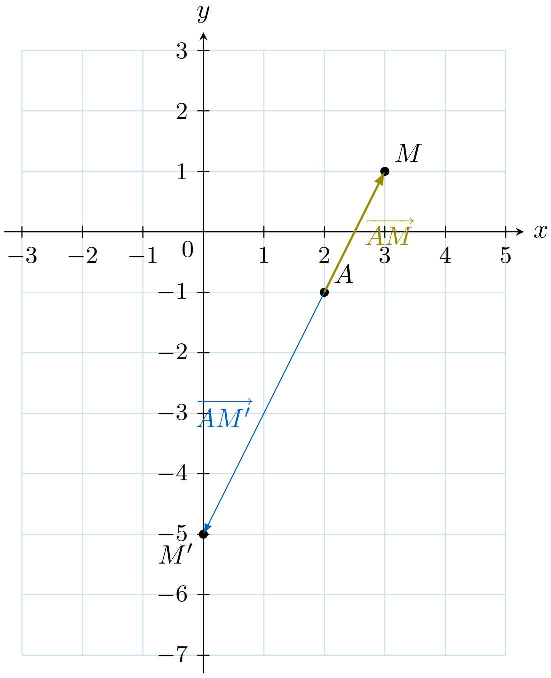
Ex 72: Let $A(2,\ -1)$ and $M(3,\ 1)$. The point M' is the image of M by the homothety of center A and ratio $k=-2$ so that $\overrightarrow{AM'}=-2\,\overrightarrow{AM}$.
Find the coordinates of M'.

$$M'=(\boxed{0},\ \boxed{-5})$$

Answer:

- $\overrightarrow{AM}=\begin{pmatrix}3-2\\1-(-1)\end{pmatrix}=\begin{pmatrix}1\\2\end{pmatrix}$
- $\overrightarrow{AM'}=-2\,\overrightarrow{AM}=-2\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}-2\\-4\end{pmatrix}$
- $$\overrightarrow{AM'}=\begin{pmatrix}x_{M'}-x_A\\y_{M'}-y_A\end{pmatrix}$$
  $$\begin{pmatrix}-2\\-4\end{pmatrix}=\begin{pmatrix}x_{M'}-2\\y_{M'}-(-1)\end{pmatrix}$$
- $x_{M'}-2=-2\implies x_{M'}=0$
- $y_{M'}-(-1)=-4\implies y_{M'}=-5$

So $M'(0,\ -5)$.



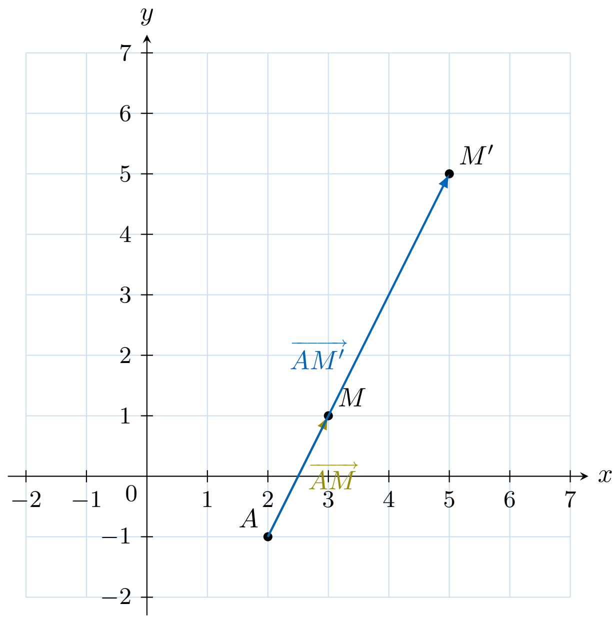
Ex 73: Let $A(2,\ -1)$ and $M(3,\ 1)$. The point M' is the image of M by the homothety of center A and ratio $k = 3$, so that $\overrightarrow{AM'} = 3\,\overrightarrow{AM}$.
Find the coordinates of M'.

$$M' = (\boxed{5}, \boxed{5})$$

Answer:

- $\overrightarrow{AM} = \begin{pmatrix} 3-2 \\ 1-(-1) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
- $\overrightarrow{AM'} = 3\,\overrightarrow{AM} = 3\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$
- $$\begin{aligned} \overrightarrow{AM'} &= \begin{pmatrix} x_{M'} - x_A \\ y_{M'} - y_A \end{pmatrix} \\ \begin{pmatrix} 3 \\ 6 \end{pmatrix} &= \begin{pmatrix} x_{M'} - 2 \\ y_{M'} - (-1) \end{pmatrix} \end{aligned}$$
- $x_{M'} - 2 = 3 \implies x_{M'} = 5$
- $y_{M'} - (-1) = 6 \implies y_{M'} = 5$

So $M'(5,\ 5)$.



G MAGNITUDE OF A VECTOR

G.1 CALCULATING THE LENGTH OF A VECTOR

Ex 74: Calculate the length of $\vec{v} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$$\|\vec{v}\| = \boxed{\sqrt{5}} \text{ units}$$

Answer:

$$\begin{aligned} \| \quad \vec{v} \quad \| &= \sqrt{2^3 + (-1)^2} \\ &= \sqrt{4+1} \\ &= \sqrt{5} \text{ units} \end{aligned}$$

Ex 75: Calculate the length of $\vec{p} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$

$$\|\vec{p}\| = \boxed{5} \text{ units}$$

Answer:

$$\begin{aligned} \| \quad \vec{p} \quad \| &= \sqrt{0^2 + (-5)^2} \\ &= \sqrt{0+25} \\ &= \sqrt{25} \\ &= 5 \text{ units} \end{aligned}$$

Ex 76: Calculate the length of $\vec{u} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

$$\|\vec{u}\| = \boxed{\sqrt{40}} \text{ units}$$

Answer:

$$\begin{aligned} \| \quad \vec{u} \quad \| &= \sqrt{(-6)^2 + 2^2} \\ &= \sqrt{36+4} \\ &= \sqrt{40} \text{ units} \end{aligned}$$

Ex 77: Calculate the length of $\vec{q} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$\|\vec{q}\| = \boxed{\sqrt{2}} \text{ units}$$

Answer:

$$\begin{aligned} \| \quad \vec{q} \quad \| &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{1+1} \\ &= \sqrt{2} \text{ units} \end{aligned}$$

G.2 CALCULATING THE DISTANCE BETWEEN TWO POINTS

Ex 78: Let $A(2,\ 3)$ and $B(7,\ -1)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{5} \\ \boxed{-4} \end{pmatrix}$$

2. Calculate the distance AB.

$$AB = \boxed{\sqrt{41}} \text{ units}$$

Answer:

1. $$\overrightarrow{AB} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \end{pmatrix} = \begin{pmatrix} 7-2 \\ -1-3 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$

2. $$AB = \| \overrightarrow{AB} \| = \sqrt{5^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$$

Ex 79: Let $A(-2,\ 5)$ and $B(4,\ 2)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{6} \\ \boxed{-3} \end{pmatrix}$$

2. Calculate the distance AB.

$$AB = \boxed{\sqrt{45}} \text{ units}$$

Answer:

1. $$\overrightarrow{AB} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \end{pmatrix} = \begin{pmatrix} 4-(-2) \\ 2-5 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$$

2. $$AB = \| \overrightarrow{AB} \| = \sqrt{6^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45}$$

Ex 80: Let $A(0,\ -2)$ and $B(-3,\ 6)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{-3} \\ \boxed{8} \end{pmatrix}$$

2. Calculate the distance AB.

$$AB = \boxed{\sqrt{73}} \text{ units}$$

Answer:

1. $$\overrightarrow{AB} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \end{pmatrix} = \begin{pmatrix} -3-0 \\ 6-(-2) \end{pmatrix} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$$

2. $$AB = \| \overrightarrow{AB} \| = \sqrt{(-3)^2 + 8^2} = \sqrt{9+64} = \sqrt{73}$$

G.3 USING COORDINATES TO DETERMINE TRIANGLE TYPES

Ex 81: Let $A(0,\ 0)$, $B(6,\ 0)$, and $C(6,\ 8)$.

1. Calculate the lengths AB, BC, and CA.
   - $AB = \boxed{6}$
   - $BC = \boxed{8}$
   - $CA = \boxed{10}$
2. Calculate the perimeter of triangle ABC.

$$\text{Perimeter} = \boxed{24} \text{ units}$$

Answer:

1.
   - $AB = \sqrt{(6-0)^2 + (0-0)^2} = \sqrt{36} = 6$
   - $BC = \sqrt{(6-6)^2 + (8-0)^2} = \sqrt{64} = 8$
   - $CA = \sqrt{(0-6)^2 + (0-8)^2} = \sqrt{36+64} = \sqrt{100} = 10$
2. Perimeter $= 6 + 8 + 10 = 24$ units

Ex 82: Let $A(0,\ 0)$, $B(4,\ 0)$, and $C(2,\ 4)$.

1. Calculate the lengths AB, BC, and CA.
   - $AB = \boxed{4}$
   - $BC = \boxed{\sqrt{20}}$
   - $CA = \boxed{\sqrt{20}}$
2. Is the triangle ABC isosceles?

Yes

Answer:

1.
   - $AB = \sqrt{(4-0)^2 + (0-0)^2} = \sqrt{16} = 4$
   - $BC = \sqrt{(2-4)^2 + (4-0)^2} = \sqrt{4+16} = \sqrt{20}$
   - $CA = \sqrt{(0-2)^2 + (0-4)^2} = \sqrt{4+16} = \sqrt{20}$
2. Yes, the triangle is isosceles because $BC = CA = \sqrt{20}$, so two sides are equal.

Ex 83: Let $A(0,\ 0)$, $B(2,\ 2\sqrt{3})$, and $C(4,\ 0)$.

1. Calculate the lengths AB, BC, and CA.
   - $AB = \boxed{4}$
   - $BC = \boxed{4}$
   - $CA = \boxed{4}$
2. Is the triangle ABC equilateral?

Yes

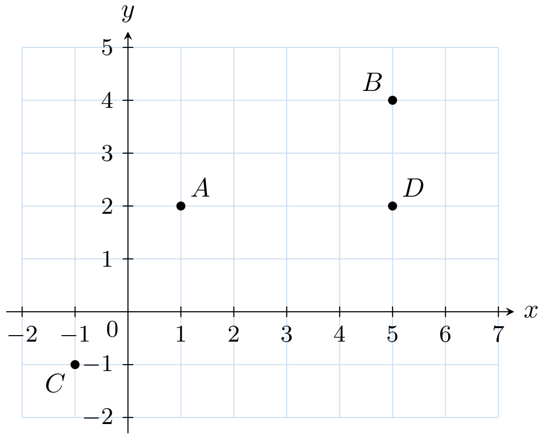
Answer:

1.
   - $AB = \sqrt{(2-0)^2 + (2\sqrt{3}-0)^2} = \sqrt{4+12} = \sqrt{16} = 4$
   - $BC = \sqrt{(4-2)^2 + (0-2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$
   - $CA = \sqrt{(0-4)^2 + (0-0)^2} = \sqrt{16} = 4$
2. Yes, the triangle is equilateral because $AB = BC = CA = 4$.

H COLINEARITY

H.1 TESTING PARALLELISM/ALIGNMENT USING VECTORS

Ex 84:



Let $A(1,\ 2)$, $B(5,\ 4)$, $C(-1,\ -1)$, and $D(5,\ 2)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{4} \\ \boxed{2} \end{pmatrix}$$

2. Calculate the vector $\overrightarrow{CD}$.

$$\overrightarrow{CD} = \begin{pmatrix} \boxed{6} \\ \boxed{3} \end{pmatrix}$$

3. Calculate the determinant $\det(\overrightarrow{AB}, \overrightarrow{CD})$.

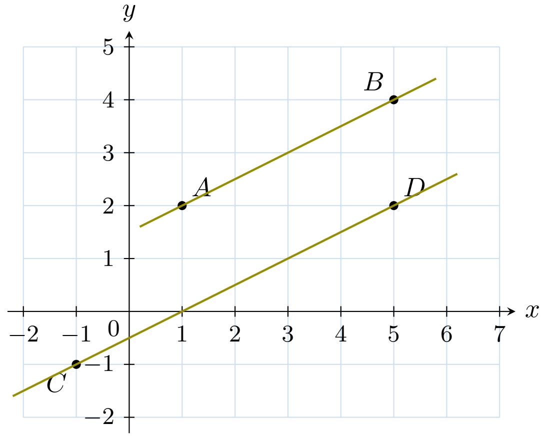
$$\det(\overrightarrow{AB}, \overrightarrow{CD}) = \boxed{0}$$

4. Are the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ parallel?

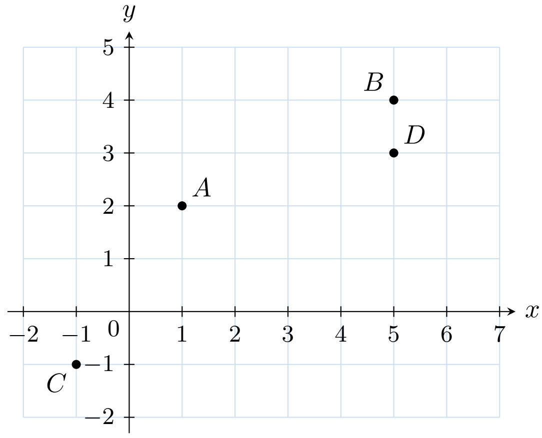
Yes

Answer:

1. $\overrightarrow{AB} = \begin{pmatrix} 5-1 \\ 4-2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$
2. $\overrightarrow{CD} = \begin{pmatrix} 5-(-1) \\ 2-(-1) \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$
3. $\det(\overrightarrow{AB}, \overrightarrow{CD}) = \begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix} = 4 \times 3 - 6 \times 2 = 12 - 12 = 0$
4. Since the determinant is zero, the vectors are colinear, so the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are **parallel**.



Ex 85:



Let $A(1,\ 2)$, $B(5,\ 4)$, $C(-1,\ -1)$, and $D(5,\ 3)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{4} \\ \boxed{2} \end{pmatrix}$$

2. Calculate the vector $\overrightarrow{CD}$.

$$\overrightarrow{CD} = \begin{pmatrix} \boxed{6} \\ \boxed{4} \end{pmatrix}$$

3. Calculate the determinant $\det(\overrightarrow{AB}, \overrightarrow{CD})$.

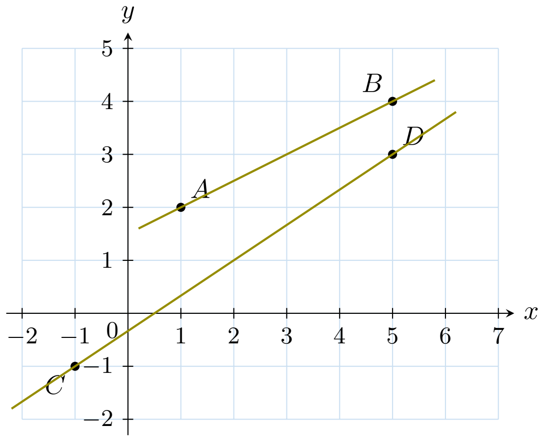
$$\det(\overrightarrow{AB}, \overrightarrow{CD}) = \boxed{4}$$

4. Are the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ parallel?

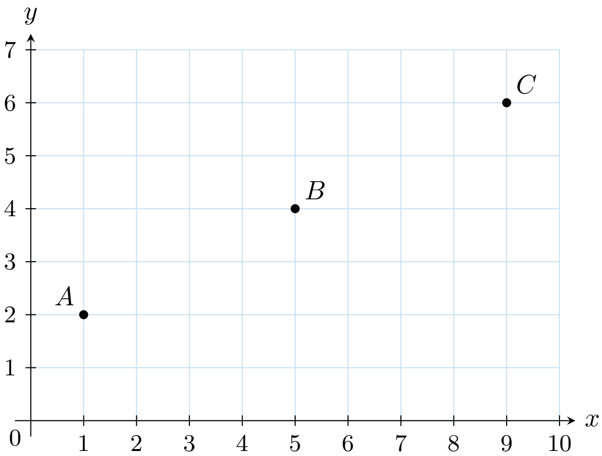
No

Answer:

1. $\overrightarrow{AB} = \begin{pmatrix} 5-1 \\ 4-2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$
2. $\overrightarrow{CD} = \begin{pmatrix} 5-(-1) \\ 3-(-1) \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$
3. $\det(\overrightarrow{AB}, \overrightarrow{CD}) = \begin{vmatrix} 4 & 6 \\ 2 & 4 \end{vmatrix} = 4 \times 4 - 6 \times 2 = 16 - 12 = 4$
4. The determinant is not zero, so the vectors are not colinear. Therefore, the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are **not parallel**.



Ex 86:



Let $A(1,\ 2)$, $B(5,\ 4)$, and $C(9,\ 6)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{4} \\ \boxed{2} \end{pmatrix}$$

2. Calculate the vector $\overrightarrow{AC}$.

$$\overrightarrow{AC} = \begin{pmatrix} \boxed{8} \\ \boxed{4} \end{pmatrix}$$

3. Calculate the determinant $\det(\overrightarrow{AB}, \overrightarrow{AC})$.

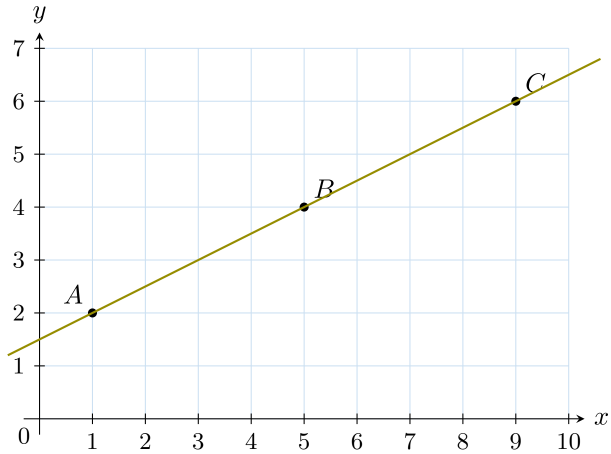
$$\det(\overrightarrow{AB}, \overrightarrow{AC}) = \boxed{0}$$

4. Are the points A, B, and C aligned?

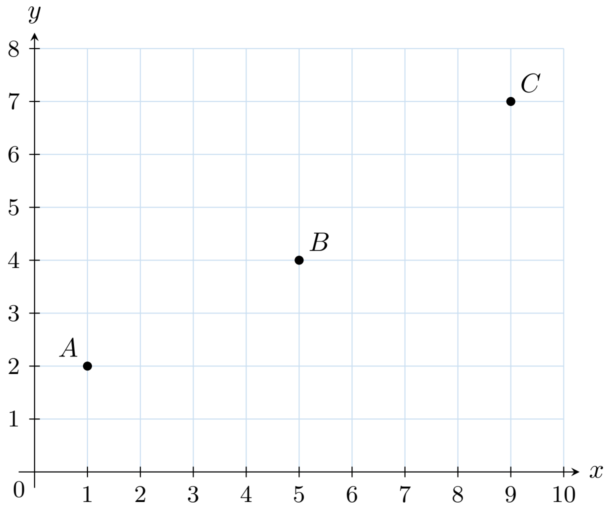
Yes

Answer:

1. $\overrightarrow{AB} = \begin{pmatrix} 5-1 \\ 4-2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$
2. $\overrightarrow{AC} = \begin{pmatrix} 9-1 \\ 6-2 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$
3. $\det(\overrightarrow{AB}, \overrightarrow{AC}) = \begin{vmatrix} 4 & 8 \\ 2 & 4 \end{vmatrix} = 4 \times 4 - 8 \times 2 = 16 - 16 = 0$
4. Since the determinant is zero, the vectors are colinear. Therefore, the points A, B, and C are aligned.



Ex 87:



Let $A(1,\ 2)$, $B(5,\ 4)$, and $C(9,\ 7)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{4} \\ \boxed{2} \end{pmatrix}$$

2. Calculate the vector $\overrightarrow{AC}$.

$$\overrightarrow{AC} = \begin{pmatrix} \boxed{8} \\ \boxed{5} \end{pmatrix}$$

3. Calculate the determinant $\det(\overrightarrow{AB}, \overrightarrow{AC})$.

$$\det(\overrightarrow{AB}, \overrightarrow{AC}) = \boxed{4}$$

4. Are the points A, B, and C aligned?

No

Answer:

1. $\overrightarrow{AB} = \begin{pmatrix} 5-1 \\ 4-2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$
2. $\overrightarrow{AC} = \begin{pmatrix} 9-1 \\ 7-2 \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \end{pmatrix}$
3. $\det(\overrightarrow{AB}, \overrightarrow{AC}) = \begin{vmatrix} 4 & 8 \\ 2 & 5 \end{vmatrix} = 4 \times 5 - 8 \times 2 = 20 - 16 = 4$

4. Since the determinant is not zero, the vectors are not colinear. Therefore, the points A, B, and C are **not aligned**.