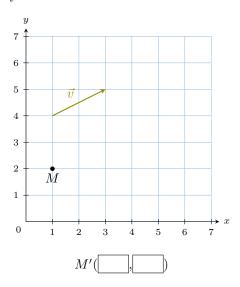
# **VECTORS**

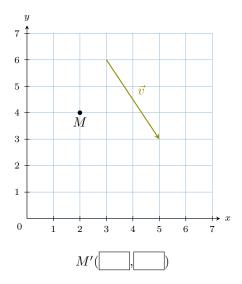
# **A DEFINITIONS**

#### A.1 FINDING THE IMAGE OF A POINT

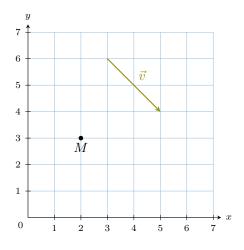
**Ex 1:** Find the coordinates of the image of point M under a translation by vector  $\vec{v}$ .



**Ex 2:** Find the coordinates of the image of point M under a translation by vector  $\vec{v}$ .

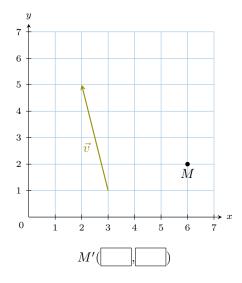


**Ex 3:** Find the coordinates of the image of point M under a translation by vector  $\vec{v}$ .

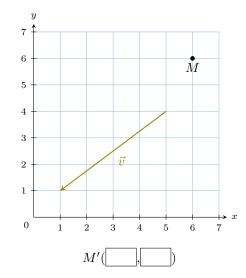




**Ex 4:** Find the coordinates of the image of point M under a translation by vector  $\vec{v}$ .

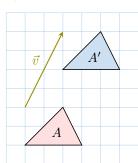


**Ex 5:** Find the coordinates of the image of point M under a translation by vector  $\vec{v}$ .



## A.2 TRANSLATION OF FIGURES

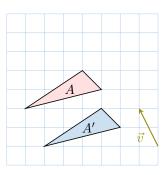
MCQ 6: Is the figure A' the image of figure A under a translation by vector  $\vec{v}$ ?



 $\square$  Yes

 $\square$  No

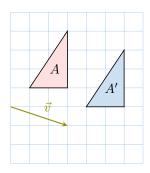
MCQ 7: Is the figure A' the image of figure A under a translation by vector  $\vec{v}$ ?



 $\square$  Yes

 $\square$  No

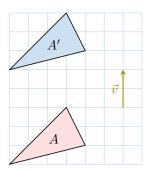
MCQ 8: translation by vector  $\vec{v}$ ?



☐ Yes

 $\square$  No

Is the figure A' the image of figure A under a MCQ 9: translation by vector  $\vec{v}$ ?

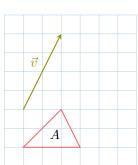


☐ Yes

 $\square$  No

## A.3 DRAWING IMAGES FIGURES

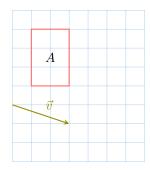
Ex 10: Draw the figure A', the image of figure A under a translation by vector  $\vec{v}$ .



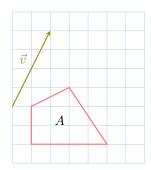
**Ex 11:** Draw the figure A', the image of figure A under a translation by vector  $\vec{v}$ .



Is the figure A' the image of figure A under a **Ex 12:** Draw the figure A', the image of figure A under a translation by vector  $\vec{v}$ .

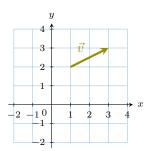


Ex 13: Draw the figure A', the image of figure A under a translation by vector  $\vec{v}$  .



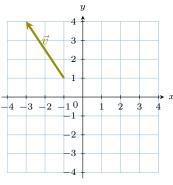
#### A.4 FINDING COMPONENTS OF A VECTOR

**Ex 14:** Find the components of the vector  $\vec{v}$ .



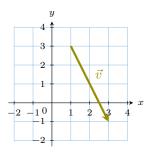
$$\vec{v} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

**Ex 15:** Find the components of the vector  $\vec{v}$ .



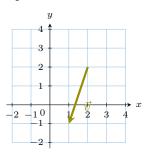
 $ec{v} = \left( egin{array}{c} \egin{array}{c} egin{array}{c} egin{array}{c} egin{array}{c} \egin{array}{c} \egin{array}{c}$ 

**Ex 16:** Find the components of the vector  $\vec{v}$ .



$$ec{v} = \left( egin{array}{c} igcap \ \end{array} 
ight)$$

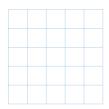
Ex 17: Find the components of the vector  $\vec{v}$ .



$$ec{v} = \left( \begin{array}{c} \\ \end{array} \right)$$

# A.5 REPRESENTING VECTORS ON A GRID

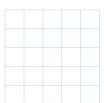
**Ex 18:** Draw the arrows diagram of the vector  $\vec{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ .



**Ex 19:** Draw the arrows diagram of the vector  $\vec{u} = \begin{pmatrix} -2\\4 \end{pmatrix}$ .



**Ex 20:** Draw the arrows diagram of the vector  $\vec{w} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ .



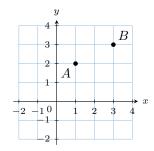
**Ex 21:** Draw the arrows diagram of the vector  $\vec{z} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$ .



# **B TWO POINT NOTATION**

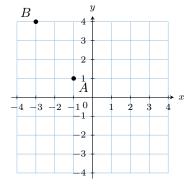
# **B.1 FINDING COMPONENTS OF A VECTOR**

**Ex 22:** Find the components of the vector  $\overrightarrow{AB}$ .



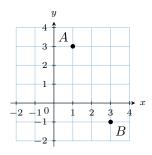
$$\overrightarrow{AB} = \left(\begin{array}{c} \\ \end{array}\right)$$

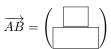
**Ex 23:** Find the components of the vector  $\overrightarrow{AB}$ .



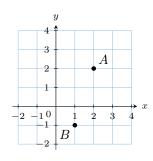
$$\overrightarrow{AB} = \left( \begin{array}{|c|} \hline \\ \hline \end{array} \right)$$

**Ex 24:** Find the components of the vector  $\overrightarrow{AB}$ .





**Ex 25:** Find the components of the vector  $\overrightarrow{AB}$ .



$$\overrightarrow{AB} = \left(\begin{array}{c} \\ \end{array}\right)$$

## **B.2 FINDING THE VECTOR COMPONENTS**

**Ex 26:** For A(1,2) and B(3,3), find the components of the vector  $\overrightarrow{AB}$ .

$$\overrightarrow{AB} = \left(\begin{array}{c} \\ \end{array}\right)$$

**Ex 27:** For E(0, 5) and F(4, 2), find the components of the vector  $\overrightarrow{EF}$ .

$$\overrightarrow{EF} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

**Ex 28:** For B(-2, 0) and C(3, 4), find the components of the vector  $\overrightarrow{BC}$ .

$$\overrightarrow{BC} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

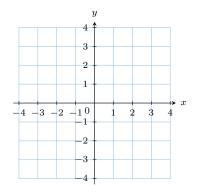
**Ex 29:** For B(3, 3) and A(1, 2), find the components of the vector  $\overrightarrow{BA}$ .

$$\overrightarrow{BA} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

#### **B.3 PLACING A POINT USING A VECTOR**

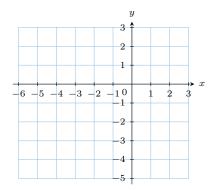
Ex 30:

- 1. Plot the point A(-4,1).
- 2. Plot the point B such that  $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ .



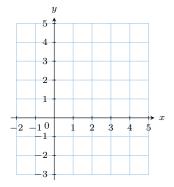
Ex 31:

- 1. Plot the point C(1, -3).
- 2. Plot the point D such that  $\overrightarrow{CD} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ .



Ex 32:

- 1. Plot the point E(0,2).
- 2. Plot the point F such that  $\overrightarrow{EF} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ .



#### **B.4 FINDING THE VECTOR COMPONENTS IN 3D**

**Ex 33:** For A(1,1,1) and B(2,3,4), find the components of the vector  $\overrightarrow{AB}$ .

$$\overrightarrow{AB} = \left(\begin{array}{|c|c|} \hline \\ \hline \end{array}\right)$$

**Ex 34:** For A(4,2,3) and B(1,4,3), find the components of the vector  $\overrightarrow{AB}$ .

$$\overrightarrow{AB} = \left(\begin{array}{c} \\ \\ \\ \end{array}\right)$$

**Ex 35:** For A(4,4,4) and B(1,3,0), find the components of the vector  $\overrightarrow{AB}$ .

$$\overrightarrow{AB} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

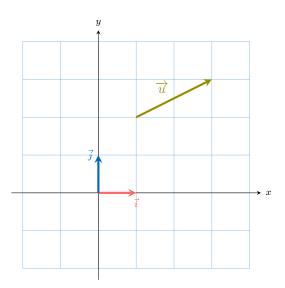
**Ex 36:** For A(2,0,-3) and B(0,3,1), find the components of the vector  $\overrightarrow{AB}$ .

$$\overrightarrow{AB} = \left( \begin{array}{c} \\ \\ \end{array} \right)$$

# C BASE VECTORS

## **C.1 DECOMPOSING A VECTOR**

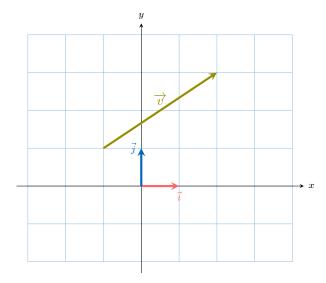
Ex 37:



Write in unit vector form:

$$\overrightarrow{u} = \overrightarrow{i} + \overrightarrow{j}$$

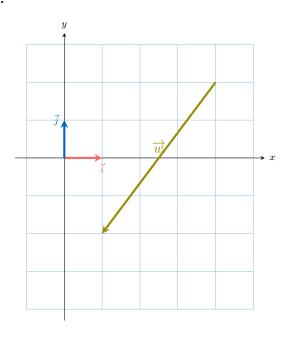
Ex 38:



Write in unit vector form:

$$\overrightarrow{v} = \overrightarrow{i} + \overrightarrow{j}$$

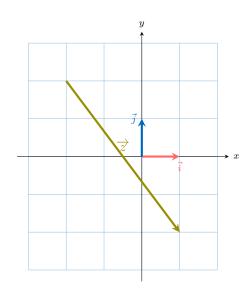
Ex 39:



Write in unit vector form:

$$\overrightarrow{w} = \overrightarrow{i} + \overrightarrow{j}$$

Ex 40:



Write in unit vector form:

$$\overrightarrow{z} = \overrightarrow{i} + \overrightarrow{j}$$

# C.2 CONVERTING COMPONENT FORM TO UNIT VECTOR FORM

 $\mathbf{Ex}\ \mathbf{41:}\ \mathbf{Write}\ \mathrm{in}\ \mathrm{unit}\ \mathrm{vector}\ \mathrm{form:}$ 

$$\binom{2}{1} = \vec{i} + \vec{j}$$

Ex 42: Write in unit vector form:

$$\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \boxed{\vec{i} - \boxed{\vec{j}}}$$

Ex 43: Write in unit vector form:

$$\begin{pmatrix} -2\\3 \end{pmatrix} = \vec{i} + \vec{j}$$

Ex 44: Write in unit vector form:

$$\begin{pmatrix} -3 \\ -1 \end{pmatrix} = \boxed{\vec{i} - \boxed{\vec{j}}}$$

# D EQUALITY BETWEEN VECTORS

## **D.1 DRAWING EQUAL VECTORS**

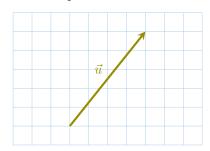
**Ex 45:** Draw a vector equal to  $\vec{v}$ .



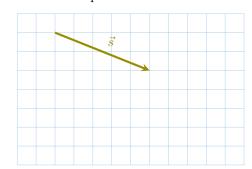
**Ex 46:** Draw a vector equal to  $\vec{w}$ .



**Ex 47:** Draw a vector equal to  $\vec{u}$ .



**Ex 48:** Draw a vector equal to  $\vec{s}$ .



# D.2 FINDING THE COORDINATES OF A POINT WITH A GIVEN VECTOR

**Ex 49:** Let A(2,3), B(5,7), and C(1,-2).

Find the coordinates of the point D such that  $\overrightarrow{AB} = \overrightarrow{CD}$ .

**Ex 50:** Let A(0,0), B(4,3), and C(2,1).

Find the coordinates of the point D such that  $\overrightarrow{AB} = \overrightarrow{CD}$ .

**Ex 51:** Let A(-1,2), B(1,5), and C(3,-1).

Find the coordinates of the point D such that  $\overrightarrow{AB} = \overrightarrow{CD}$ .

## **D.3 SOLVING VECTOR EQUATIONS**

**Ex 52:** Determine the values of x and y for the following vector equality:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$
$$x = \boxed{\qquad} \text{and } y = \boxed{\qquad}$$

**Ex 53:** Determine the values of x and y for the following vector equality:

$$x\overrightarrow{i} + y\overrightarrow{j} = -2\overrightarrow{i} + \overrightarrow{j}$$
  
 $x =$  and  $y =$ 

**Ex 54:** Determine the values of x and y for the following vector equality:

$$\begin{pmatrix} x \\ y+1 \end{pmatrix} = \begin{pmatrix} 2x-1 \\ 3-x \end{pmatrix}$$
$$x = \boxed{\quad \text{and } y = \boxed{\quad}}$$

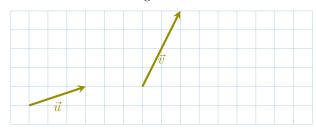
**Ex 55:** Determine the values of x and y for the following vector equality:

$$x\overrightarrow{i} + y\overrightarrow{j} = 2y\overrightarrow{i} + 3\overrightarrow{j}$$
  
 $x =$  and  $y =$ 

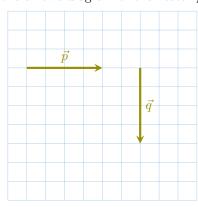
# **E VECTOR ADDITION AND SUBTRACTION**

#### **E.1 DRAWING THE SUM OF TWO VECTORS**

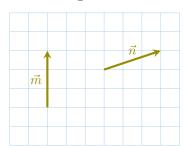
**Ex 56:** Draw the arrows diagram of the vector  $\vec{u} + \vec{v}$ .



**Ex 57:** Draw the arrows diagram of the vector  $\vec{p} + \vec{q}$ .



**Ex 58:** Draw the arrows diagram of the vector  $\vec{m} + \vec{n}$ .



# **E.2 CALCULATING THE SUM OF VECTORS**

**Ex 59:** Calculate the sum of the vectors  $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$  and  $\vec{b} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$  $\binom{-5}{4}$ .

**Ex 60:** Calculate the sum of the vectors  $\vec{u} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$  and  $\vec{v} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$  $\binom{-1}{5}$ .

$$\vec{u} + \vec{v} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

 $\binom{8}{-4}$ .

$$\vec{p} + \vec{q} = \left( \begin{array}{c} \\ \\ \end{array} \right)$$

**Ex 62:** Calculate the sum of the vectors  $\vec{m} = \begin{pmatrix} 0 \\ -7 \end{pmatrix}$  and  $\vec{n} = \begin{pmatrix} 0 \\ -7 \end{pmatrix}$  $\binom{5}{3}$ .

$$\vec{m} + \vec{n} = \left( \begin{array}{c} \boxed{\phantom{a}} \end{array} \right)$$

## **E.3 RECOGNIZING SUMS OF VECTORS**

MCQ 63: Calculate the sum of vectors:  $\overrightarrow{AB} + \overrightarrow{BC}$ .

- $\Box \overrightarrow{CA}$
- $\square \overrightarrow{AC}$
- $\Box \overrightarrow{BA}$
- $\Box \overrightarrow{CB}$

Calculate the sum of vectors:  $\overrightarrow{BC} + \overrightarrow{AB}$ . MCQ 64:

- $\Box \overrightarrow{CB}$
- $\Box \overrightarrow{BA}$

- $\Box \overrightarrow{0}$
- $\square \overrightarrow{AC}$

Calculate the sum of vectors:  $\overrightarrow{AB} + \overrightarrow{BA}$ MCQ 65:

- $\Box \overrightarrow{BA}$
- $\Box \overrightarrow{AB}$
- $\Box \overrightarrow{0}$

Calculate the sum of vectors:  $\overrightarrow{EA} + \overrightarrow{AB} + \overrightarrow{BC}$ . MCQ 66:

- $\Box \overrightarrow{CE}$
- $\Box \overrightarrow{0}$
- $\square \overrightarrow{AC}$
- $\square \overrightarrow{EC}$

Calculate the sum of vectors:  $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$ . MCQ 67:

- $\Box \overrightarrow{AD}$
- $\Box \overrightarrow{DA}$
- $\square \overrightarrow{BD}$
- $\Box \overrightarrow{0}$

# **E.4 CALCULATING THE SUM OF VECTORS IN 3D**

**Ex 61:** Calculate the sum of the vectors  $\vec{p} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$  and  $\vec{q} =$ **Ex 68:** Calculate the sum of the vectors  $\vec{a} = \begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix}$  and  $\vec{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$  $\begin{pmatrix} -5\\4\\2 \end{pmatrix}$ .

$$\vec{a} + \vec{b} = \begin{pmatrix} \boxed{\phantom{a}} \\ \boxed{\phantom{a}} \end{pmatrix}$$

**Ex 69:** Calculate the sum of the vectors  $\vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$  and  $\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ 

$$\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$
.

$$\vec{u} + \vec{v} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

**Ex 70:** Calculate the sum of the vectors  $\vec{m} = \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix}$  and  $\vec{n} = \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix}$ 

$$\begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}$$
.

$$ec{m} + ec{n} = \left( egin{array}{c} ec{m} \end{array} 
ight)$$

**Ex 71:** Calculate the sum of the vectors  $\vec{p} = \begin{pmatrix} 10 \\ -8 \\ 6 \end{pmatrix}$  and  $\vec{q} = \begin{pmatrix} \mathbf{Ex 78:} \\ \mathbf{Ex 78:} \\ \mathbf{Ex 78:} \\ \mathbf{Calculate the negative of the vector } \vec{p} = \begin{pmatrix} 0 \\ -8 \end{pmatrix}.$ 

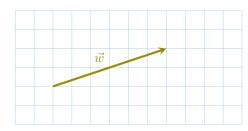
$$\vec{p} + \vec{q} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

## **E.5 DRAWING THE NEGATIVE OF A VECTOR**

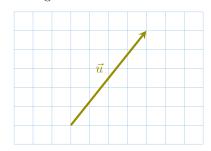
**Ex 72:** Draw the negative vector of  $\vec{v}$ .



**Ex 73:** Draw the negative vector of  $\vec{w}$ .



**Ex 74:** Draw the negative vector of  $\vec{u}$ .



#### **E.6 CALCULATING THE NEGATIVE OF A VECTOR**

**Ex 75:** Calculate the negative of the vector  $\vec{a} = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$ .

$$-\vec{a} = \left( \begin{array}{c} \\ \\ \end{array} \right)$$

**Ex 76:** Calculate the negative of the vector  $\vec{b} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$ .

$$-ec{b} = \left( egin{array}{c} lacksquare \ \end{array} 
ight)$$

**Ex 77:** Calculate the negative of the vector  $\vec{u} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ .

$$-\vec{u} = \left(\begin{array}{c} \\ \end{array}\right)$$

$$-\vec{p} = \left(\begin{array}{|c|c|} \hline \end{array}\right)$$

# **E.7 CALCULATING THE DIFFERENCE OF VECTORS**

**Ex 79:** Calculate the difference of the vectors  $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$  and  $\vec{b} = \begin{pmatrix} -5\\4 \end{pmatrix}$ .

$$ec{a}-ec{b}=\left( egin{array}{c} \egin{array}{c} \egin{array}{c} \egin{array}{c} \egin{array}{c} \egin{array$$

**Ex 80:** Calculate the difference of the vectors  $\vec{u} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$  and  $\vec{v} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$ .

$$\vec{u} - \vec{v} = \left(\begin{array}{c} \hline \\ \hline \end{array}\right)$$

**Ex 81:** Calculate the difference of the vectors  $\vec{p} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$  and  $\vec{q} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}.$ 

$$ec{p}-ec{q}=\left( egin{array}{c} \egin{array}{c} \egin{array$$

**Ex 82:** Calculate the difference of the vectors  $\vec{m} = \begin{pmatrix} 0 \\ -7 \end{pmatrix}$  and  $\vec{n} = {5 \choose 3}.$ 

$$ec{m}-ec{n}=\left( egin{array}{c} ec{m} \end{array} 
ight)$$

#### **DRAWING** THE SUBTRACTION **TWO VECTORS**

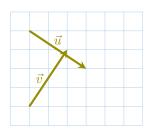
**Ex 83:** Draw the vector of  $\vec{u} - \vec{v}$ . (Do that on your graph paper.)



**Ex 84:** Draw the vector of  $\vec{u} - \vec{v}$ . (Do that on your graph paper.)



**Ex 85:** Draw the vector of  $\vec{u} - \vec{v}$ . (Do that on your graph paper.)



# E.9 CALCULATING THE DIFFERENCE OF VECTORS IN 3D

**Ex 86:** Calculate the difference of the vectors  $\vec{a} = \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}$  and

$$\vec{b} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}.$$

$$ec{a}-ec{b}=egin{pmatrix} oxedsymbol{oxedsymbol{\Box}} oxedsymbol{\Box} oxendow{\Box} oxeta oxendow{\Box} oxeta oxendow{\Box} oxeta oxendow{\Box} oxendow{\Box} oxendow{\Box} oxendow{\Box} oxendow{\Box} oxeta oxendow{\Box} oxendow{\Box} oxendow{\Box} oxendow{\Box} oxendow{\Box} oxendow{\Box} oxendow{\Box} oxendow{\Box} oxeta oxendow{\Box} oxeta oxextbox{\Box} oxendow{\Box} oxendow{\Box} oxendow{\Box} oxeta oxendow{\Box} oxendow{\Box} oxeta oxextbox{\Box} oxeta oxeta oxextbox{\Box} oxeta oxeta oxextbox{\Box} oxeta oxextbox{\Box} oxeta oxextbox{\Box} oxeta oxextbox{\Box} oxeta oxeta oxextbox{\Box} oxeta oxextbox{\Box} oxe$$

**Ex 87:** Calculate the difference of the vectors  $\vec{c} = \begin{pmatrix} -6 \\ 7 \\ -1 \end{pmatrix}$  and

$$\vec{d} = \begin{pmatrix} -2\\ -3\\ 5 \end{pmatrix}.$$

$$\vec{c} - \vec{d} = \begin{pmatrix} & & \\ & & \end{pmatrix}$$

**Ex 88:** Calculate the difference of the vectors  $\vec{e} = \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}$  and

$$\vec{f} = \begin{pmatrix} 5 \\ 0 \\ -3 \end{pmatrix}.$$

$$\vec{e} - \vec{f} = \left(\begin{array}{|c|c|} \hline \\ \hline \end{array}\right)$$

## F SCALAR MULTIPLICATION

#### F.1 MULTIPLYING A VECTOR BY A SCALAR

**Ex 89:** Calculate the product of the vector  $\vec{b} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$  by 3.

$$3\vec{b} = \left(\begin{array}{|c|c|} \hline \\ \hline \end{array}\right)$$

**Ex 90:** Calculate the product of the vector  $\vec{u} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$  by -2.

$$-2\vec{u} = \left(\begin{array}{|c|c|} \hline \\ \hline \end{array}\right)$$

**Ex 91:** Calculate the product of the vector  $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$  by -4.

$$-4\vec{a} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

**Ex 92:** Calculate the product of the vector  $\vec{p} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$  by 0.5.

$$\frac{1}{2}\vec{p} = \left( \boxed{\phantom{1}} \right)$$

# F.2 CALCULATING LINEAR COMBINATIONS OF VECTORS

**Ex 93:** Calculate  $3\vec{a} - \vec{b}$  where  $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$  and  $\vec{b} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$ .

$$3\vec{a} - \vec{b} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

**Ex 94:** Calculate  $2(\vec{u}+2\vec{v})$  where  $\vec{u}=\begin{pmatrix}1\\-2\end{pmatrix}$  and  $\vec{v}=\begin{pmatrix}3\\5\end{pmatrix}$ .

$$2(\vec{u} + 2\vec{v}) = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

**Ex 95:** Calculate  $4\vec{p} - 2\vec{q}$  where  $\vec{p} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$  and  $\vec{q} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ .

$$4\vec{p} - 2\vec{q} = \begin{pmatrix} \boxed{\phantom{0}} \\ \boxed{\phantom{0}} \end{pmatrix}$$

**Ex 96:** Calculate  $-3\vec{u} + 5\vec{v}$  where  $\vec{u} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$  and  $\vec{v} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ .

$$-3\vec{u} + 5\vec{v} = \left(\begin{array}{|c|} \\ \hline \end{array}\right)$$

# F.3 DETERMINING THE IMAGE OF A POINT UNDER A HOMOTHETY

**Ex 97:** Let O(0,0) and M(3,-2). The point M' is the image of M by the homothety of center O and ratio k=2 so that  $2\overrightarrow{OM} = \overrightarrow{OM'}$ .

Find the coordinates of M'.

**Ex 98:** Let A(2,-1) and M(3,1). The point M' is the image of M by the homothety of center A and ratio k=-2 so that  $\overrightarrow{AM'}=-2\overrightarrow{AM}$ .

Find the coordinates of M'.

**Ex 99:** Let A(2,-1) and M(3,1). The point M' is the image of M by the homothety of center A and ratio k=3, so that  $\overrightarrow{AM'}=3\overrightarrow{AM}$ .

Find the coordinates of M'.

# F.4 CALCULATING LINEAR COMBINATIONS OF VECTORS IN 3D

# **Ex 100:** Calculate $3\vec{a} - \vec{b}$ where $\vec{a} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -5 \\ 4 \\ -2 \end{pmatrix}$ .

$$3\vec{a} - \vec{b} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

**Ex 101:** Calculate 
$$2\vec{u} + 4\vec{v}$$
 where  $\vec{u} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}$  and  $\vec{v} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$ .

$$2\vec{u} + 4\vec{v} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

**Ex 102:** Calculate 
$$5\vec{p} - 2\vec{q}$$
 where  $\vec{p} = \begin{pmatrix} -1\\2\\-2 \end{pmatrix}$  and  $\vec{q} = \begin{pmatrix} 4\\-3\\0 \end{pmatrix}$ .

$$5\vec{p} - 2\vec{q} = \left(\begin{array}{c} \\ \\ \\ \end{array}\right)$$

# G MAGNITUDE AND UNIT VECTORS

#### G.1 CALCULATING THE LENGTH OF A VECTOR

**Ex 103:** Calculate the length of  $\overrightarrow{v} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ 

$$\|\overrightarrow{v}\| = \boxed{\quad \text{units}}$$

**Ex 104:** Calculate the length of  $\overrightarrow{p} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$ 

$$\|\overrightarrow{p}\| = \boxed{\phantom{a}}$$
 units

**Ex 105:** Calculate the length of  $\overrightarrow{u} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$ 

$$\|\overrightarrow{u}\| = \boxed{\quad \text{units}}$$

**Ex 106:** Calculate the length of  $\overrightarrow{q} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ 

$$\|\overrightarrow{q}\| = \boxed{\text{units}}$$

# G.2 CALCULATING THE DISTANCE BETWEEN TWO POINTS

**Ex 107:** Let A(2,3) and B(7,-1).

1. Calculate the vector  $\overrightarrow{AB}$ .

$$\overrightarrow{AB} = \left( \begin{array}{c} \\ \\ \end{array} \right)$$

2. Calculate the distance AB.

$$AB = \boxed{\phantom{AB}}$$
 units

**Ex 108:** Let A(-2,5) and B(4,2).

1. Calculate the vector  $\overrightarrow{AB}$ .

$$\overrightarrow{AB} = \left( \begin{array}{c} \hline \\ \hline \end{array} \right)$$

2. Calculate the distance AB.

$$AB = \boxed{\qquad}$$
 units

**Ex 109:** Let A(0, -2) and B(-3, 6).

1. Calculate the vector  $\overrightarrow{AB}$ .

$$\overrightarrow{AB} = \left( \begin{array}{c} \\ \\ \end{array} \right)$$

2. Calculate the distance AB.

$$AB =$$
 units

# G.3 USING COORDINATES TO DETERMINE TRIANGLE TYPES

**Ex 110:** Let A(0,0), B(6,0), and C(6,8).

1. Calculate the lengths AB, BC, and CA.

• 
$$CA =$$

2. Calculate the perimeter of triangle ABC.

**Ex 111:** Let A(0,0), B(4,0), and C(2,4).

1. Calculate the lengths AB, BC, and CA.

•	CA =	

2. Is the triangle ABC isosceles?

$$\square$$
 No

**Ex 112:** Let A(0,0),  $B(2,2\sqrt{3})$ , and C(4,0).

1. Calculate the lengths AB, BC, and CA.

- *AB* =
- BC =
- *CA* =

2. Is the triangle ABC equilateral?

- $\square$  Yes
- $\square$  No

# G.4 CALCULATING THE LENGTH OF A VECTOR IN 3D

**Ex 113:** Calculate the length of  $\overrightarrow{v} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$ 

$$\|\overrightarrow{v}\| = \prod \text{units}$$

**Ex 114:** Calculate the length of  $\overrightarrow{u} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$ 

$$\|\overrightarrow{u}\| = \boxed{\text{units}}$$

**Ex 115:** Calculate the length of  $\overrightarrow{w} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$ 

$$\|\overrightarrow{w}\| = \boxed{\quad \text{units}}$$

## **G.5 NORMALIZING A VECTOR**

**Ex 116:** Normalize the vector  $\overrightarrow{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ .

Ex :	117:	Norma	alize th	e vecto	$r \overrightarrow{u} =$	$\binom{3}{4}$ .		

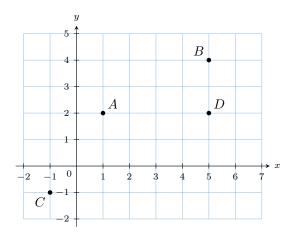
<b>Ex 118:</b> Normalize the vector $\overrightarrow{w} = \begin{pmatrix} -5\\ 2 \end{pmatrix}$
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<b>Ex 119:</b> Normalize the vector $\overrightarrow{p} = ($	$\begin{pmatrix} 0 \\ -7 \end{pmatrix}$	
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## H PARALLEL VECTORS

H.1 TESTING PARALLELISM/ALIGNMENT USING VECTORS

Ex 120:



Let A(1,2), B(5,4), C(-1,-1), and D(5,2).

1. Calculate the vector  $\overrightarrow{AB}$ .

$$\overrightarrow{AB} = \left(\begin{array}{c} \\ \end{array}\right)$$

2. Calculate the vector  $\overrightarrow{CD}$ .

$$\overrightarrow{CD} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

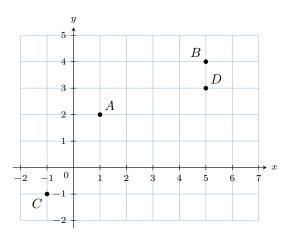
3. Calculate the determinant  $\det(\overrightarrow{AB},\,\overrightarrow{CD}).$ 

4. Are the lines  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$  parallel?

□ Yes

 $\square$  No

Ex 121:



Let A(1,2), B(5,4), C(-1,-1), and D(5,3).

1. Calculate the vector  $\overrightarrow{AB}$ .

$$\overrightarrow{AB} = \left(\begin{array}{c} \\ \end{array}\right)$$

2. Calculate the vector  $\overrightarrow{CD}$ .

$$\overrightarrow{CD} = \left(\begin{array}{|c|c|} \hline \\ \hline \end{array}\right)$$

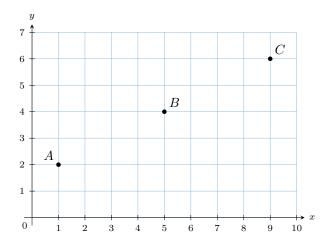
3. Calculate the determinant  $\det(\overrightarrow{AB}, \overrightarrow{CD})$ .

4. Are the lines  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$  parallel?

□ Yes

□ No

Ex 122:



Let A(1,2), B(5,4), and C(9,6).

1. Calculate the vector  $\overrightarrow{AB}$ .

$$\overrightarrow{AB} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

2. Calculate the vector  $\overrightarrow{AC}$ .

$$\overrightarrow{AC} = \left(\begin{array}{|c|} \hline \\ \hline \end{array}\right)$$

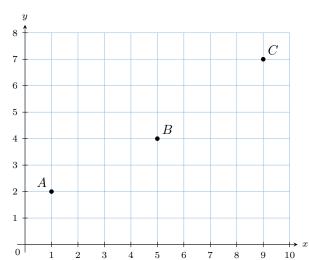
3. Calculate the determinant  $\det(\overrightarrow{AB}, \overrightarrow{AC})$ .

4. Are the points A, B, and C aligned?

□ Yes

 $\square$  No

Ex 123:



Let A(1,2), B(5,4), and C(9,7).

1. Calculate the vector  $\overrightarrow{AB}$ .

$$\overrightarrow{AB} = \left(\begin{array}{c} \\ \end{array}\right)$$

2. Calculate the vector  $\overrightarrow{AC}$ .

$$\overrightarrow{AC} = \left(\begin{array}{c} \\ \\ \end{array}\right)$$

3. Calculate the determinant  $\det(\overrightarrow{AB}, \overrightarrow{AC})$ .

$$\det(\overrightarrow{AB},\,\overrightarrow{AC}) = \boxed{\phantom{AB}}$$

- 4. Are the points A, B, and C aligned?
  - $\square$  Yes
  - $\square$  No