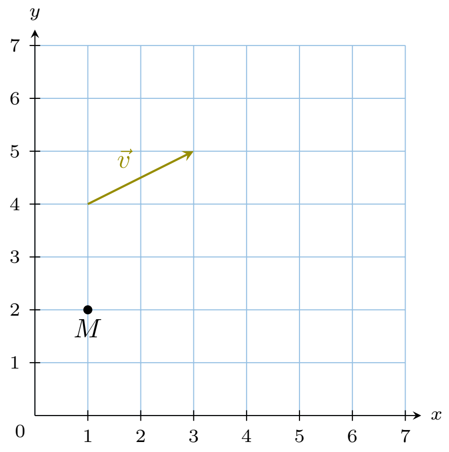
VECTORS

A DEFINITIONS

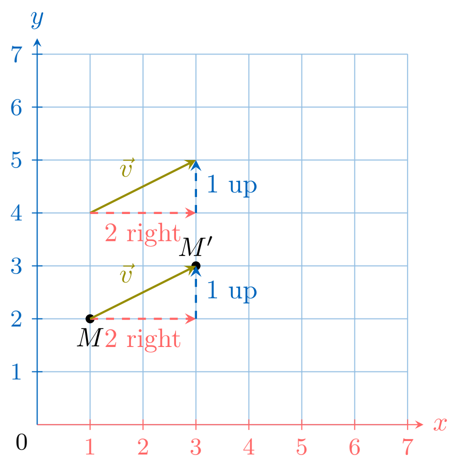
A.1 FINDING THE IMAGE OF A POINT

Ex 1: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



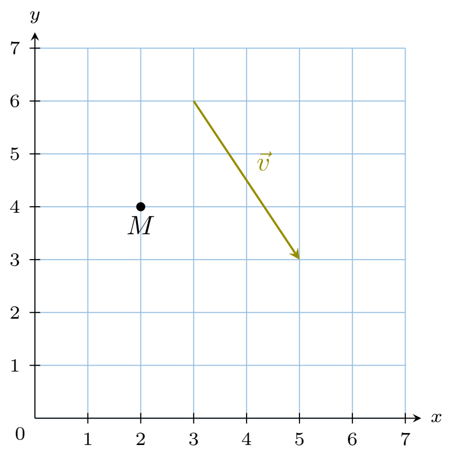
$M'(\boxed{3},\boxed{3})$

Answer:



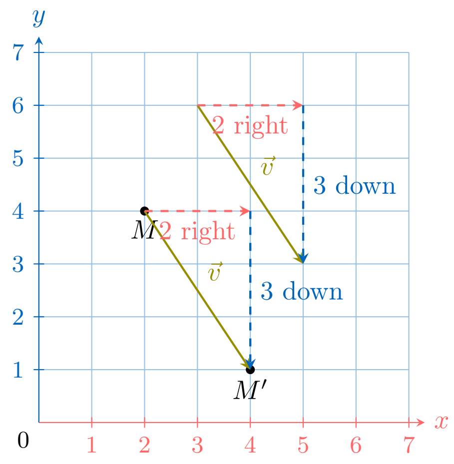
$M'(3,3)$

Ex 2: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



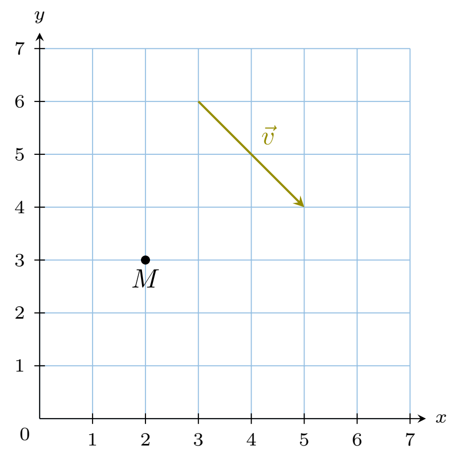
$M'(\boxed{4},\boxed{1})$

Answer:



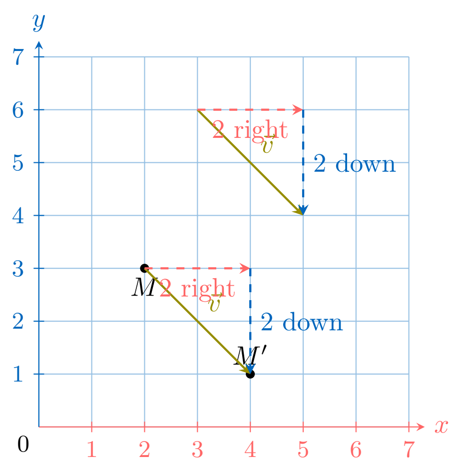
$M'(4,1)$

Ex 3: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



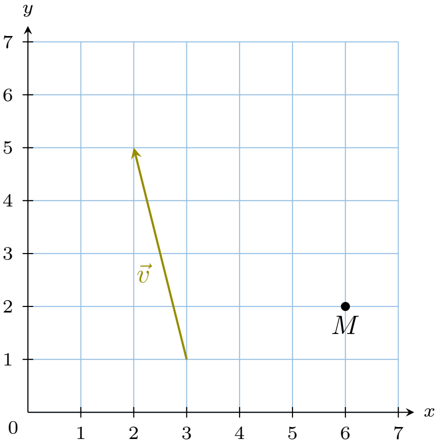
$M'(\boxed{4},\boxed{1})$

Answer:



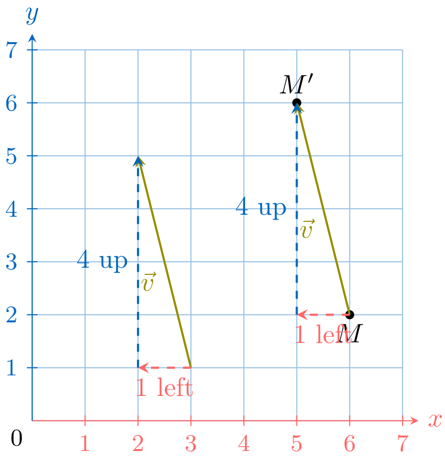
$M'(4,1)$

Ex 4: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



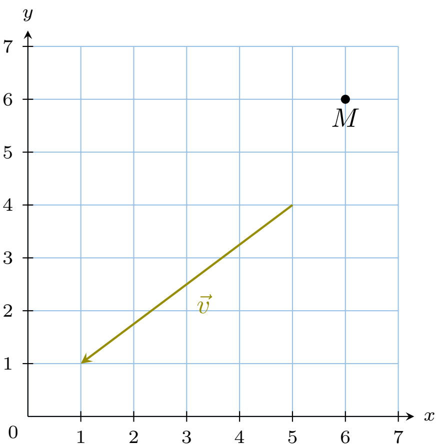
$M'(\boxed{5},\boxed{6})$

Answer:



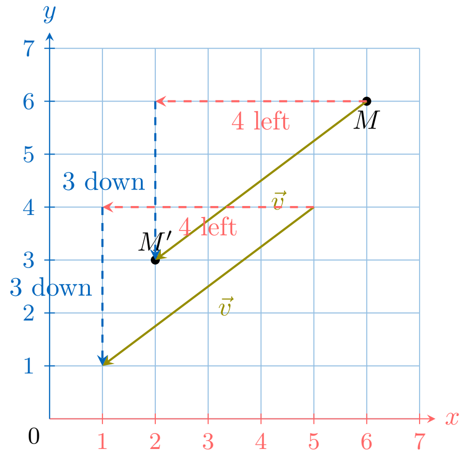
$M'(5,6)$

Ex 5: Find the coordinates of the image of point M under a translation by vector $\vec{v}$.



$M'(\boxed{2},\boxed{3})$

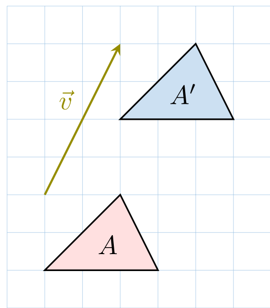
Answer:



$M'(2,3)$

A.2 TRANSLATION OF FIGURES

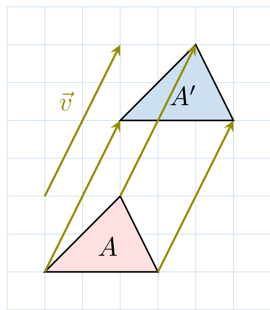
MCQ 6: Is the figure A' the image of figure A under a translation by vector $\vec{v}$?



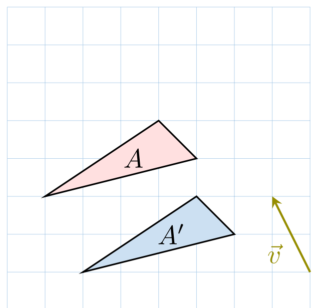
☒ Yes

☐ No

Answer: Yes



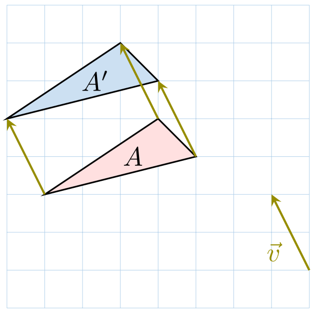
MCQ 7: Is the figure A' the image of figure A under a translation by vector $\vec{v}$?



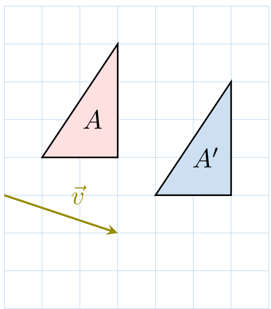
☐ Yes

☒ No

Answer: No, the figure A' is misplaced. Here is where it should be.



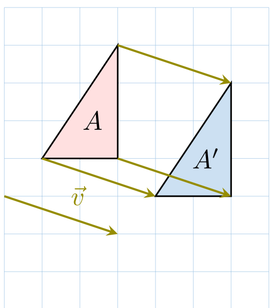
MCQ 8: Is the figure A' the image of figure A under a translation by vector $\vec{v}$?



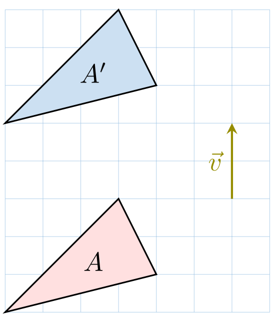
☒ Yes

☐ No

Answer: Yes



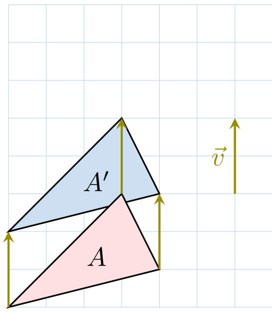
MCQ 9: Is the figure A' the image of figure A under a translation by vector $\vec{v}$?



☐ Yes

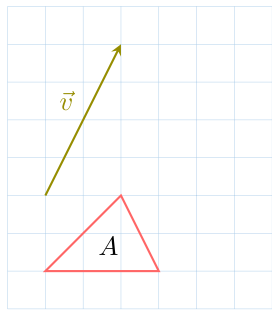
☒ No

Answer: No, the figure A' is misplaced. Here is where it should be.



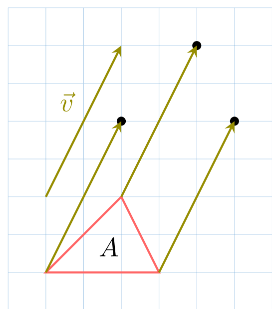
A.3 DRAWING IMAGES FIGURES

Ex 10: Draw the figure A', the image of figure A under a translation by vector $\vec{v}$.

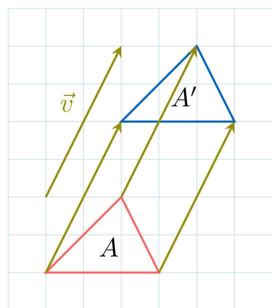


Answer:

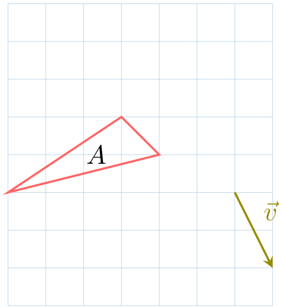
1. **Draw the image vertices**: For each vertex, translate it by the vector $\vec{v}$ by moving 2 units right and 4 units up from its original position. Place the new points on the grid.



2. **Draw the image figure**: Connect the image vertices with lines in the same order as the original figure.

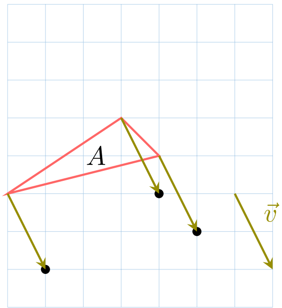


Ex 11: Draw the figure A', the image of figure A under a translation by vector $\vec{v}$.

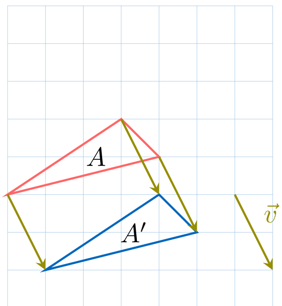


Answer:

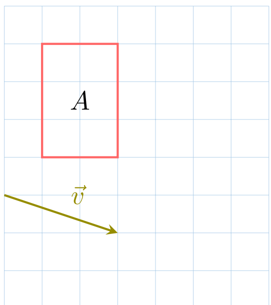
1. **Draw the image vertices**: For each vertex, translate it by the vector $\vec{v}$ by moving 1 unit right and 2 units down from its original position. Place the new points on the grid.



2. **Draw the image figure**: Connect the image vertices with lines in the same order as the original figure.

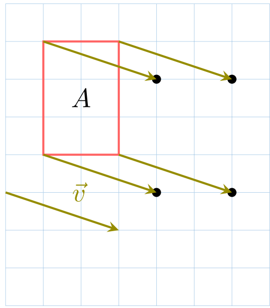


Ex 12: Draw the figure A', the image of figure A under a translation by vector $\vec{v}$.

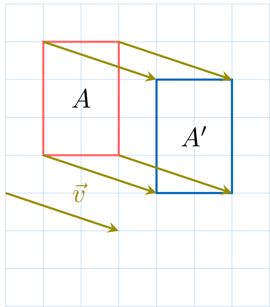


Answer:

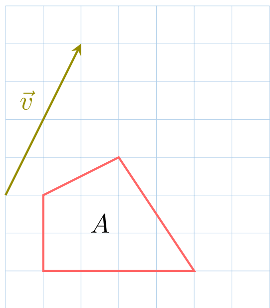
1. **Draw the image vertices**: For each vertex, translate it by the vector $\vec{v}$ by moving 3 units right and 1 unit down from its original position. Place the new points on the grid.



2. **Draw the image figure**: Connect the image vertices with lines in the same order as the original figure.

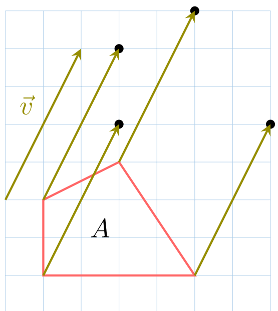


Ex 13: Draw the figure A', the image of figure A under a translation by vector $\vec{v}$.

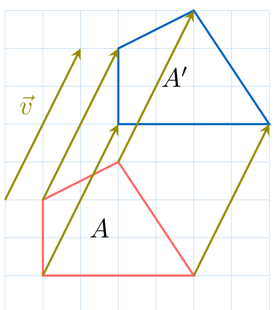


Answer:

1. **Draw the image vertices**: For each vertex, translate it by the vector $\vec{v}$ by moving 2 units right and 4 units up from its original position. Place the new points on the grid.

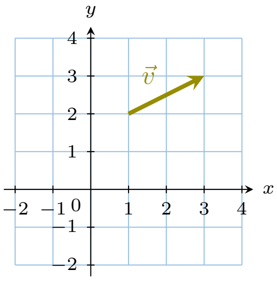


2. **Draw the image figure**: Connect the image vertices with lines in the same order as the original figure.



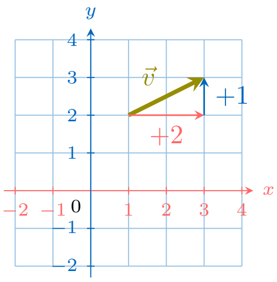
A.4 FINDING COMPONENTS OF A VECTOR

Ex 14: Find the components of the vector $\vec{v}$.



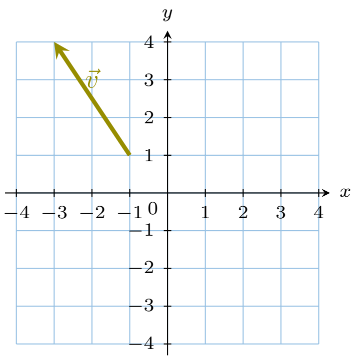
$$\vec{v} = \begin{pmatrix} \boxed{2} \\ \boxed{1} \end{pmatrix}$$

Answer:



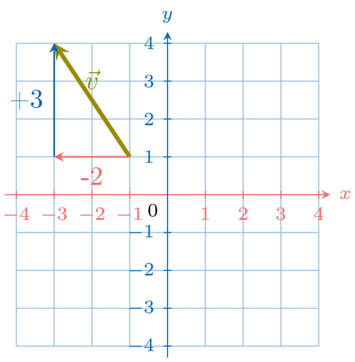
$$\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Ex 15: Find the components of the vector $\vec{v}$.



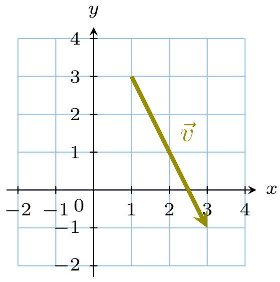
$$\vec{v} = \begin{pmatrix} \boxed{-2} \\ \boxed{3} \end{pmatrix}$$

Answer:



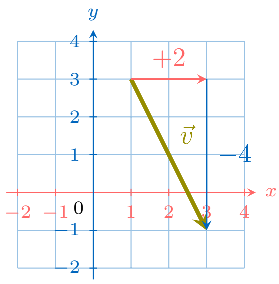
$$\vec{v} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

Ex 16: Find the components of the vector $\vec{v}$.



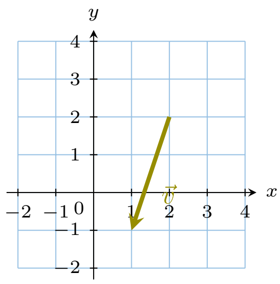
$$\vec{v} = \begin{pmatrix} \boxed{2} \\ \boxed{-4} \end{pmatrix}$$

Answer:



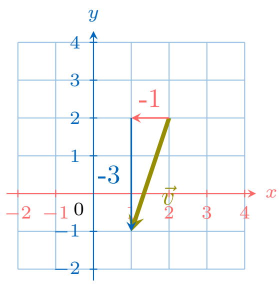
$$\vec{v} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

Ex 17: Find the components of the vector $\vec{v}$.



$$\vec{v} = \begin{pmatrix} \boxed{-1} \\ \boxed{-3} \end{pmatrix}$$

Answer:

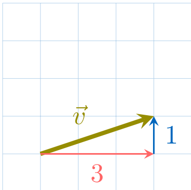


$$\vec{v} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$$

A.5 REPRESENTING VECTORS ON A GRID

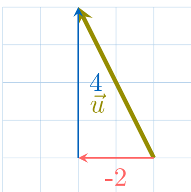
Ex 18: Draw the arrows diagram of the vector $\vec{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

Answer:



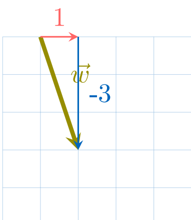
Ex 19: Draw the arrows diagram of the vector $\vec{u} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

Answer:



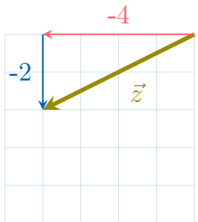
Ex 20: Draw the arrows diagram of the vector $\vec{w} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$.

Answer:



Ex 21: Draw the arrows diagram of the vector $\vec{z} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$.

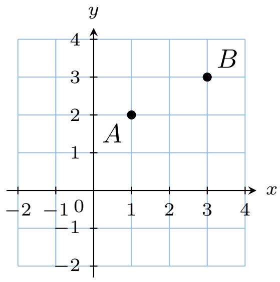
Answer:



B TWO POINT NOTATION

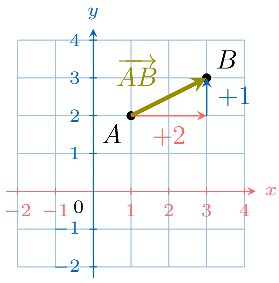
B.1 FINDING COMPONENTS OF A VECTOR

Ex 22: Find the components of the vector $\overrightarrow{AB}$.



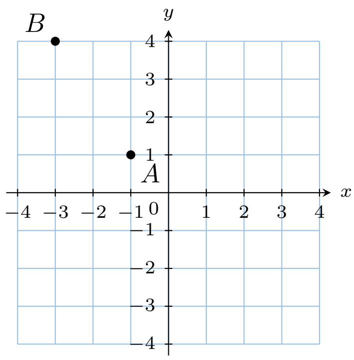
$$\overrightarrow{AB} = \begin{pmatrix} \boxed{2} \\ \boxed{1} \end{pmatrix}$$

Answer:



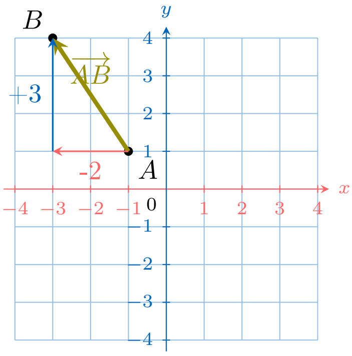
$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Ex 23: Find the components of the vector $\overrightarrow{AB}$.



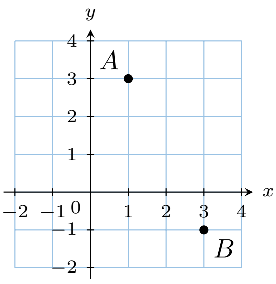
$$\overrightarrow{AB} = \begin{pmatrix}\boxed{-2}\\ \boxed{3}\end{pmatrix}$$

Answer:



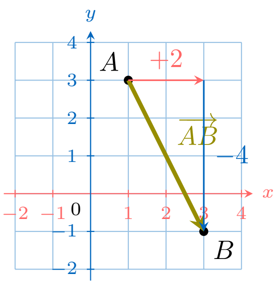
$$\overrightarrow{AB} = \begin{pmatrix}-2\\ 3\end{pmatrix}$$

Ex 24: Find the components of the vector $\overrightarrow{AB}$.



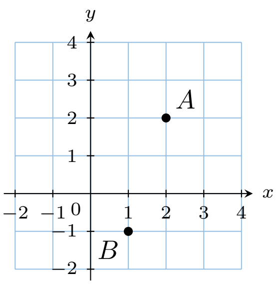
$$\overrightarrow{AB} = \begin{pmatrix}\boxed{2}\\ \boxed{-4}\end{pmatrix}$$

Answer:



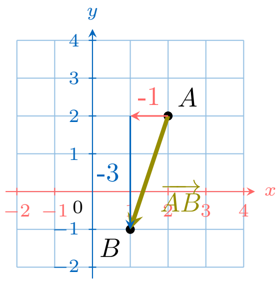
$$\overrightarrow{AB} = \begin{pmatrix}2\\ -4\end{pmatrix}$$

Ex 25: Find the components of the vector $\overrightarrow{AB}$.



$$\overrightarrow{AB} = \begin{pmatrix}\boxed{-1}\\ \boxed{-3}\end{pmatrix}$$

Answer:



$$\overrightarrow{AB} = \begin{pmatrix}-1\\ -3\end{pmatrix}$$

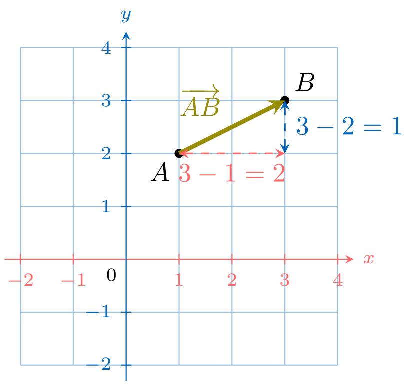
B.2 FINDING THE VECTOR COMPONENTS

Ex 26: For $A(1,2)$ and $B(3,3)$, find the components of the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix}\boxed{2}\\ \boxed{1}\end{pmatrix}$$

Answer:

$$\begin{aligned}\overrightarrow{AB} &= \begin{pmatrix}x_B - x_A\\ y_B - y_A\end{pmatrix}\\ &= \begin{pmatrix}3-1\\ 3-2\end{pmatrix}\\ &= \begin{pmatrix}2\\ 1\end{pmatrix}\end{aligned}$$

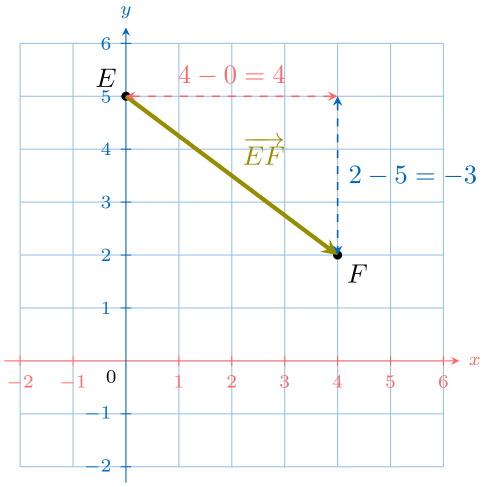


Ex 27: For $E(0, 5)$ and $F(4, 2)$, find the components of the vector $\overrightarrow{EF}$.

$$\overrightarrow{EF} = \begin{pmatrix}\boxed{4}\\ \boxed{-3}\end{pmatrix}$$

Answer:

$$\begin{aligned}\overrightarrow{EF} &= \begin{pmatrix}x_F - x_E\\ y_F - y_E\end{pmatrix}\\ &= \begin{pmatrix}4-0\\ 2-5\end{pmatrix}\\ &= \begin{pmatrix}4\\ -3\end{pmatrix}\end{aligned}$$

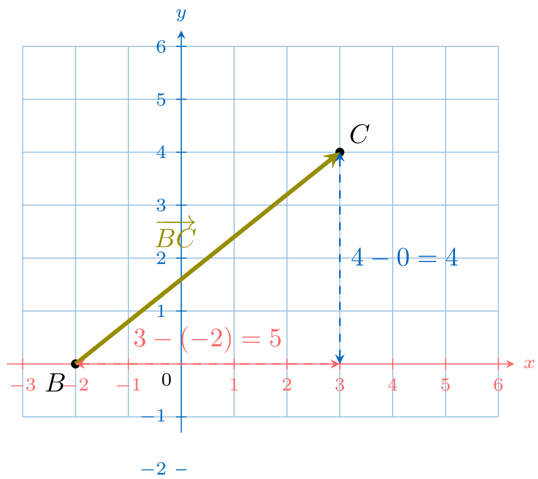


Ex 28: For $B(-2,\ 0)$ and $C(3,\ 4)$, find the components of the vector $\overrightarrow{BC}$.

$$\overrightarrow{BC} = \begin{pmatrix} \boxed{5} \\ \boxed{4} \end{pmatrix}$$

Answer:

$$\begin{aligned} \overrightarrow{BC} &= \begin{pmatrix} x_C - x_B \\ y_C - y_B \end{pmatrix} \\ &= \begin{pmatrix} 3 - (-2) \\ 4 - 0 \end{pmatrix} \\ &= \begin{pmatrix} 5 \\ 4 \end{pmatrix} \end{aligned}$$

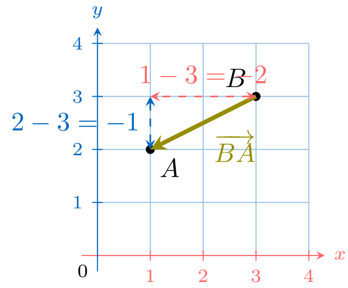


Ex 29: For $B(3,\ 3)$ and $A(1,\ 2)$, find the components of the vector $\overrightarrow{BA}$.

$$\overrightarrow{BA} = \begin{pmatrix} \boxed{-2} \\ \boxed{-1} \end{pmatrix}$$

Answer:

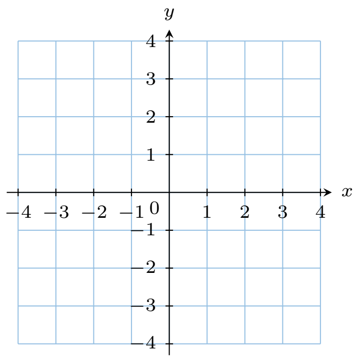
$$\begin{aligned} \overrightarrow{BA} &= \begin{pmatrix} x_A - x_B \\ y_A - y_B \end{pmatrix} \\ &= \begin{pmatrix} 1 - 3 \\ 2 - 3 \end{pmatrix} \\ &= \begin{pmatrix} -2 \\ -1 \end{pmatrix} \end{aligned}$$



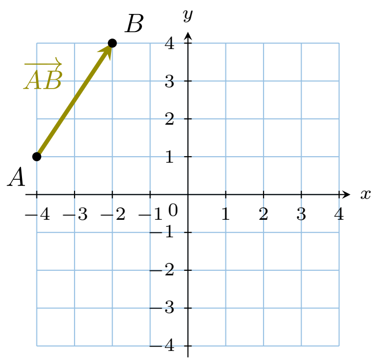
B.3 PLACING A POINT USING A VECTOR

Ex 30:

1. Plot the point $A(-4, 1)$.
2. Plot the point B such that $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.



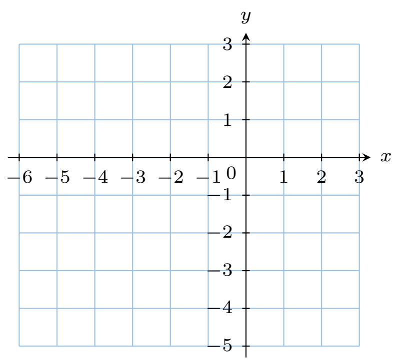
Answer:



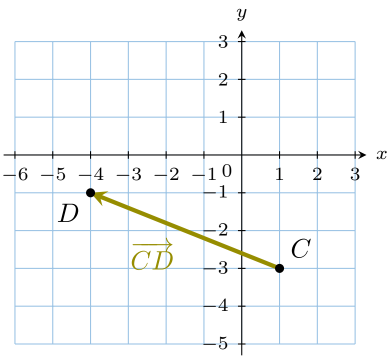
$A(-4, 1)$ and $B(-2, 4)$.

Ex 31:

1. Plot the point $C(1, -3)$.
2. Plot the point D such that $\overrightarrow{CD} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$.



Answer:

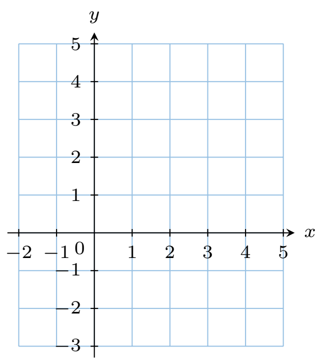


$C(1;\,-3)$ and $D(-4;\,-1)$.

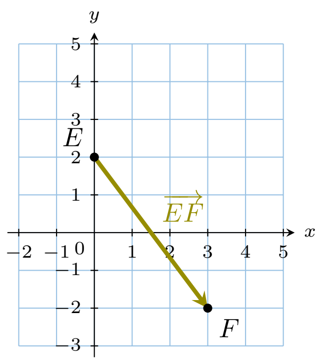
Ex 32:

1. Plot the point $E(0, 2)$.

2. Plot the point F such that $\overrightarrow{EF} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$.



Answer:



$E(0;\,2)$ and $F(3;\,-2)$.

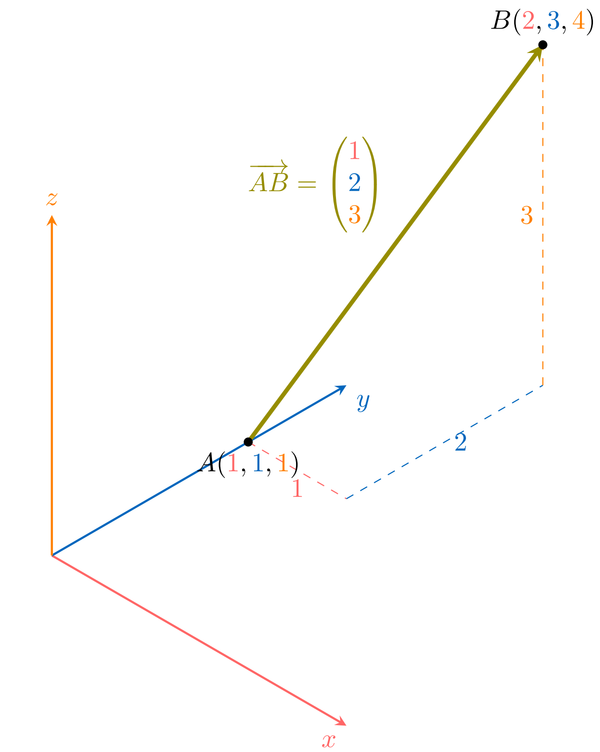
B.4 FINDING THE VECTOR COMPONENTS IN 3D

Ex 33: For $A(1, 1, 1)$ and $B(2, 3, 4)$, find the components of the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{1} \\ \boxed{2} \\ \boxed{3} \end{pmatrix}$$

Answer:

$$\begin{aligned}\overrightarrow{AB} &= \begin{pmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{pmatrix} \\ &= \begin{pmatrix} 2-1 \\ 3-1 \\ 4-1 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\end{aligned}$$

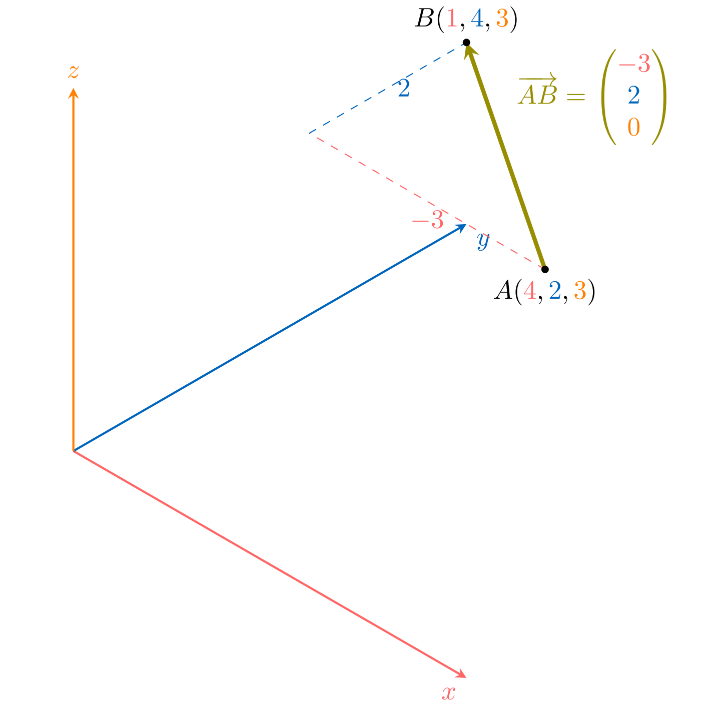


Ex 34: For $A(4, 2, 3)$ and $B(1, 4, 3)$, find the components of the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{-3} \\ \boxed{2} \\ \boxed{0} \end{pmatrix}$$

Answer:

$$\begin{aligned}\overrightarrow{AB} &= \begin{pmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{pmatrix} \\ &= \begin{pmatrix} 1-4 \\ 4-2 \\ 3-3 \end{pmatrix} \\ &= \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}\end{aligned}$$

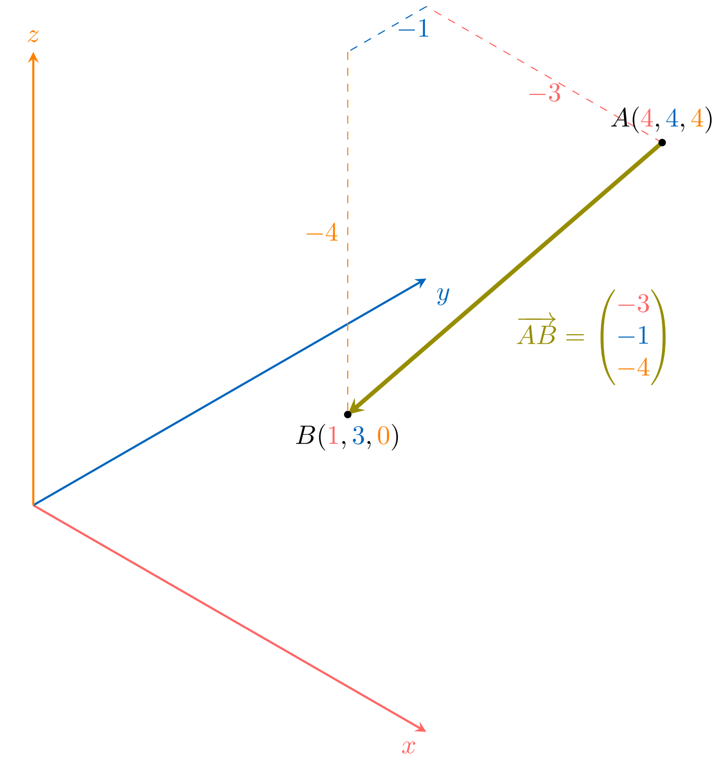


Ex 35: For $A(4, 4, 4)$ and $B(1, 3, 0)$, find the components of the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{-3} \\ \boxed{-1} \\ \boxed{-4} \end{pmatrix}$$

Answer:

$$\begin{aligned} \overrightarrow{AB} &= \begin{pmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{pmatrix} \\ &= \begin{pmatrix} 1-4 \\ 3-4 \\ 0-4 \end{pmatrix} \\ &= \begin{pmatrix} -3 \\ -1 \\ -4 \end{pmatrix} \end{aligned}$$

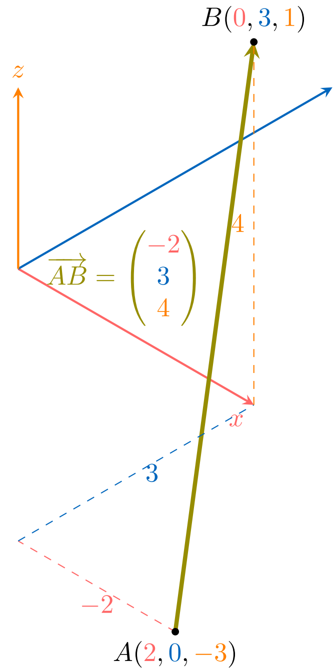


Ex 36: For $A(2, 0, -3)$ and $B(0, 3, 1)$, find the components of the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{-2} \\ \boxed{3} \\ \boxed{4} \end{pmatrix}$$

Answer:

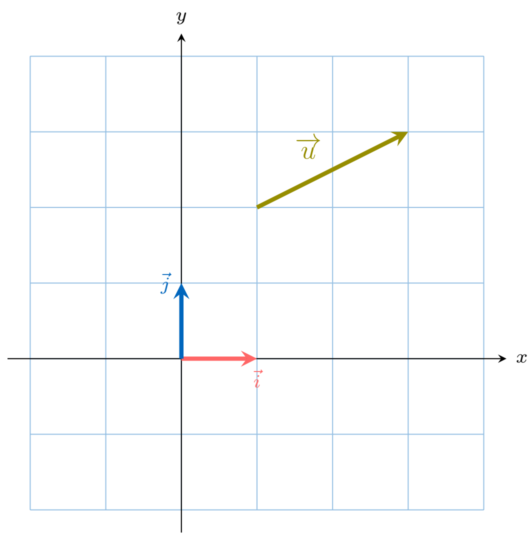
$$\begin{aligned} \overrightarrow{AB} &= \begin{pmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{pmatrix} \\ &= \begin{pmatrix} 0-2 \\ 3-0 \\ 1-(-3) \end{pmatrix} \\ &= \begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix} \end{aligned}$$



C BASE VECTORS

C.1 DECOMPOSING A VECTOR

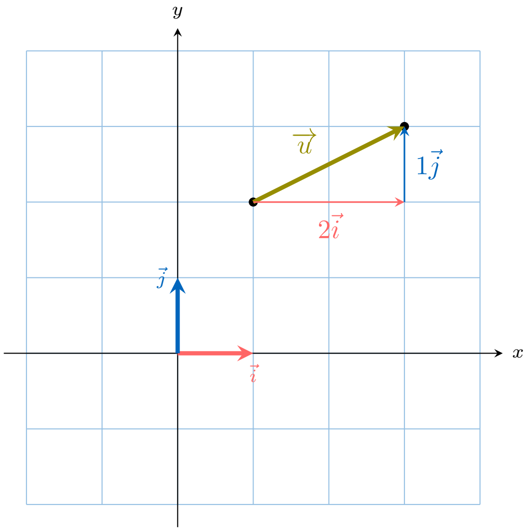
Ex 37:



Write in unit vector form:

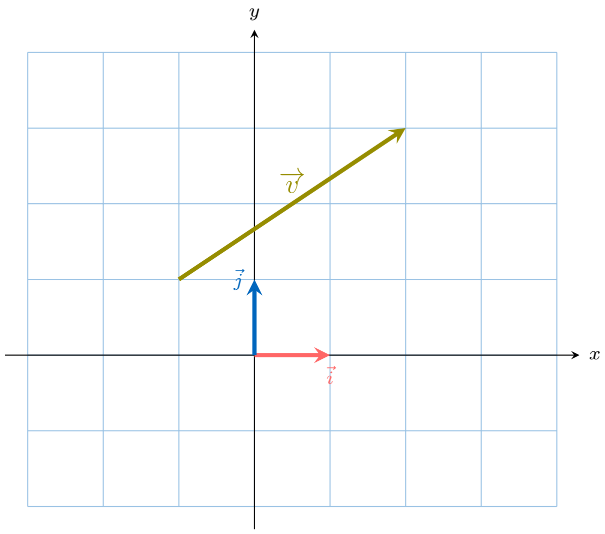
$$\vec{u} = \boxed{2}\,\vec{i} + \boxed{1}\,\vec{j}$$

Answer:



$$\vec{u} = 2\vec{i} + 1\vec{j}$$

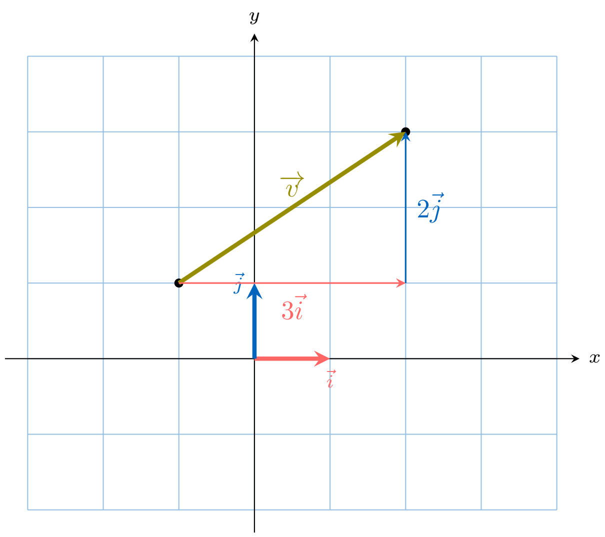
Ex 38:



Write in unit vector form:

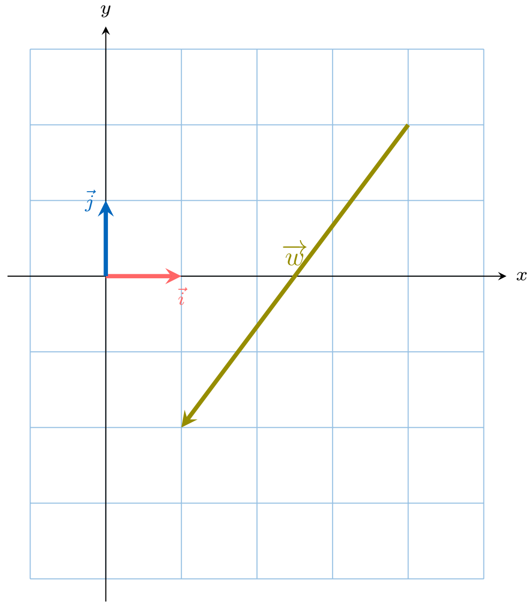
$$\vec{v} = \boxed{3}\,\vec{i} + \boxed{2}\,\vec{j}$$

Answer:



$$\vec{v} = 3\vec{i} + 2\vec{j}$$

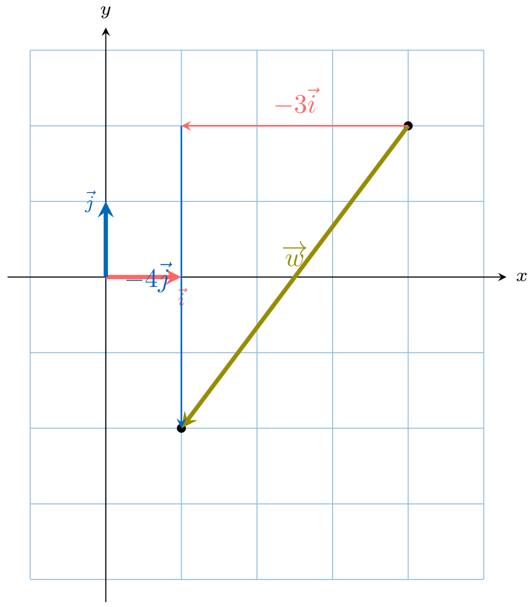
Ex 39:



Write in unit vector form:

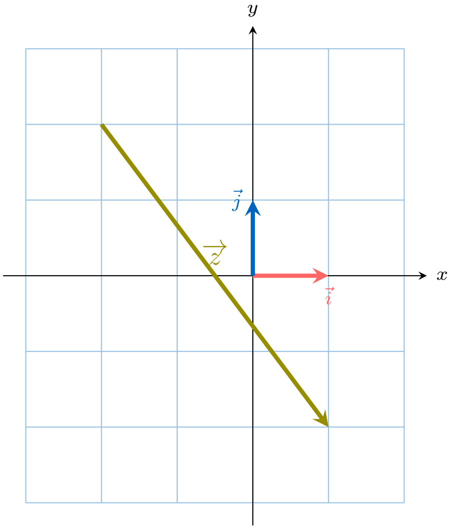
$$\vec{w} = \boxed{-3}\,\vec{i} + \boxed{-4}\,\vec{j}$$

Answer:



$$\vec{w} = -3\vec{i} - 4\vec{j}$$

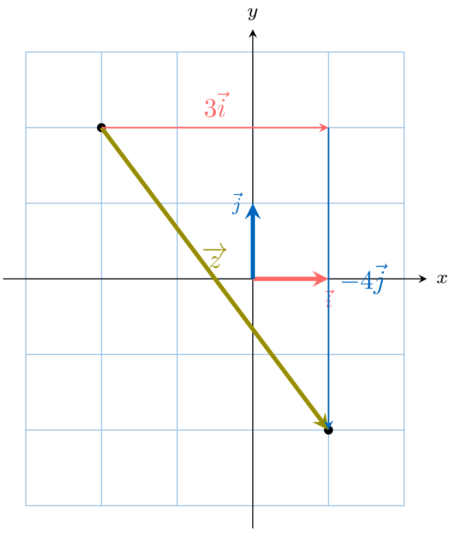
Ex 40:



Write in unit vector form:

$$\vec{z} = \boxed{3}\,\vec{i} + \boxed{-4}\,\vec{j}$$

Answer:



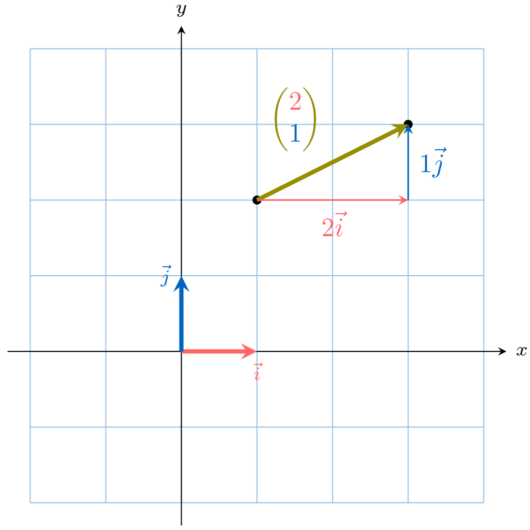
$$\vec{z} = 3\vec{i} - 4\vec{j}$$

C.2 CONVERTING COMPONENT FORM TO UNIT VECTOR FORM

Ex 41: Write in unit vector form:

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \boxed{2}\,\vec{i} + \boxed{1}\,\vec{j}$$

Answer:

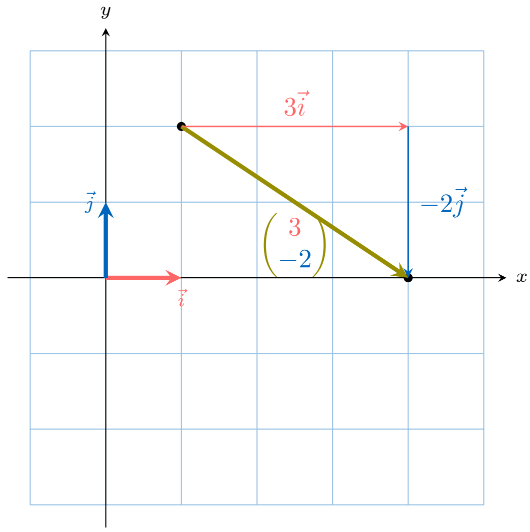


$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2\vec{i} + 1\vec{j}$$

Ex 42: Write in unit vector form:

$$\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \boxed{3}\,\vec{i} - \boxed{2}\,\vec{j}$$

Answer:

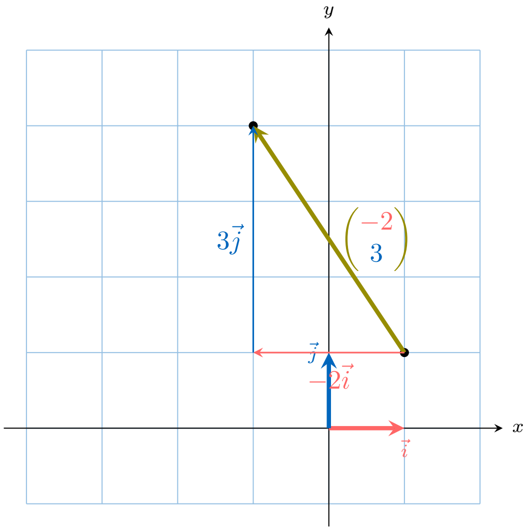


$$\begin{pmatrix} 3 \\ -2 \end{pmatrix} = 3\vec{i} - 2\vec{j}$$

Ex 43: Write in unit vector form:

$$\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \boxed{-2}\,\vec{i} + \boxed{3}\,\vec{j}$$

Answer:

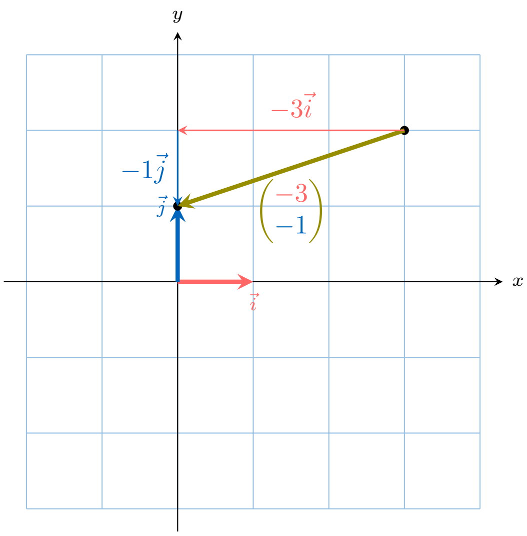


$$\begin{pmatrix}-2\\3\end{pmatrix} = -2\vec{i} + 3\vec{j}$$

Ex 44: Write in unit vector form:

$$\begin{pmatrix}-3\\-1\end{pmatrix} = \boxed{-3}\,\vec{i} - \boxed{1}\,\vec{j}$$

Answer:

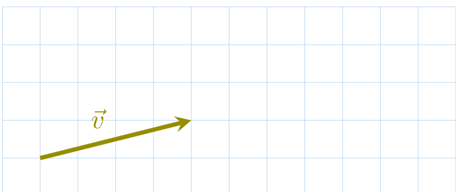


$$\begin{pmatrix}-3\\-1\end{pmatrix} = -3\vec{i} - 1\vec{j}$$

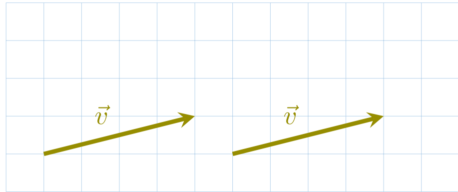
D EQUALITY BETWEEN VECTORS

D.1 DRAWING EQUAL VECTORS

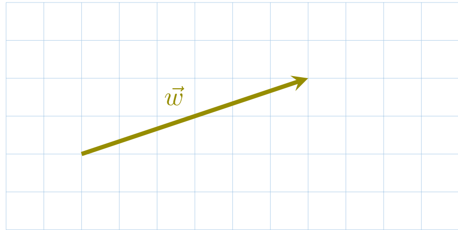
Ex 45: Draw a vector equal to $\vec{v}$.



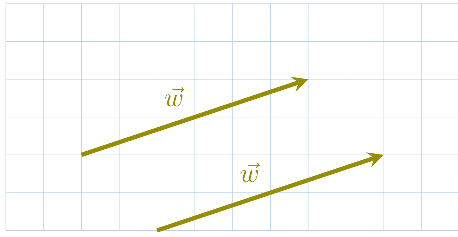
Answer: Draw a vector with the same direction, sense, and length as $\vec{v}$, starting from any point on the grid. For example:



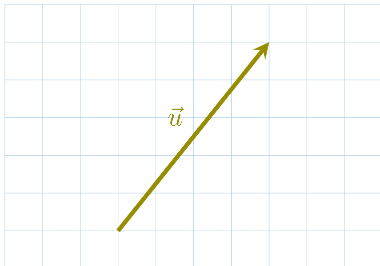
Ex 46: Draw a vector equal to $\vec{w}$.



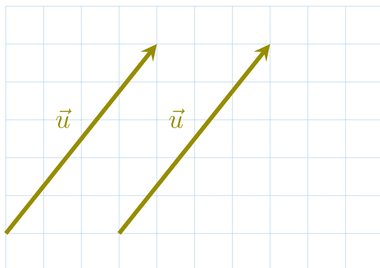
Answer: Draw a vector with the same direction, sense, and length as $\vec{w}$, starting from any point on the grid. For example:



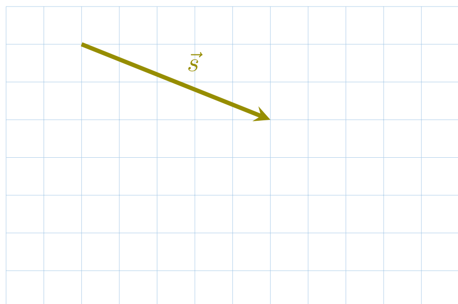
Ex 47: Draw a vector equal to $\vec{u}$.



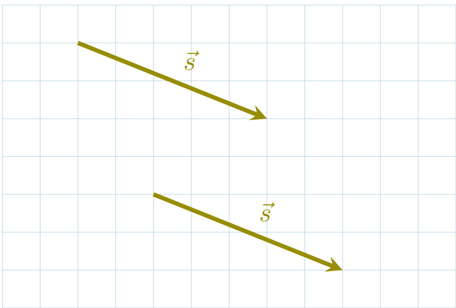
Answer: Draw a vector with the same direction, sense, and length as $\vec{u}$, starting from any point on the grid. For example:



Ex 48: Draw a vector equal to $\vec{s}$.



Answer: Draw a vector with the same direction, sense, and length as $\vec{s}$, starting from any point on the grid. For example:



D.2 FINDING THE COORDINATES OF A POINT WITH A GIVEN VECTOR

Ex 49: Let $A(2,3)$, $B(5,7)$, and $C(1,-2)$.
Find the coordinates of the point D such that $\overrightarrow{AB} = \overrightarrow{CD}$.

$$D = (\boxed{4}, \boxed{2})$$

Answer:

- First, compute the vectors:

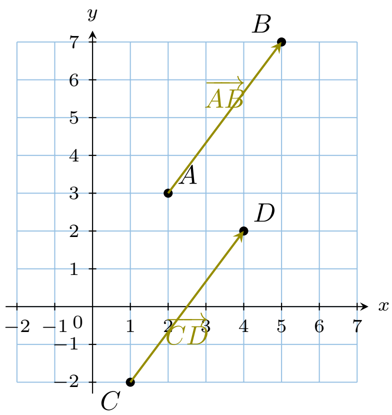
$$\overrightarrow{AB} = \begin{pmatrix} 5-2 \\ 7-3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\overrightarrow{CD} = \begin{pmatrix} x_D - 1 \\ y_D - (-2) \end{pmatrix} = \begin{pmatrix} x_D - 1 \\ y_D + 2 \end{pmatrix}$$

- Then, solve the equation:

$$\overrightarrow{AB} = \overrightarrow{CD}$$
$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} x_D - 1 \\ y_D + 2 \end{pmatrix}$$
$$3 = x_D - 1 \text{ and } 4 = y_D + 2$$
$$x_D = 3 + 1 \text{ and } y_D = 4 - 2$$
$$x_D = 4 \text{ and } y_D = +2$$

So, $D(4,2)$.



Ex 50: Let $A(0,0)$, $B(4,3)$, and $C(2,1)$.
Find the coordinates of the point D such that $\overrightarrow{AB} = \overrightarrow{CD}$.

$$D = (\boxed{6}, \boxed{4})$$

Answer:

- First, compute the vectors:

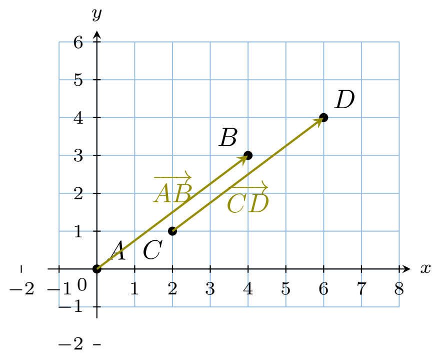
$$\overrightarrow{AB} = \begin{pmatrix} 4-0 \\ 3-0 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$\overrightarrow{CD} = \begin{pmatrix} x_D - 2 \\ y_D - 1 \end{pmatrix} = \begin{pmatrix} x_D - 2 \\ y_D - 1 \end{pmatrix}$$

- Then, solve the equation:

$$\overrightarrow{AB} = \overrightarrow{CD}$$
$$\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} x_D - 2 \\ y_D - 1 \end{pmatrix}$$
$$4 = x_D - 2 \text{ and } 3 = y_D - 1$$
$$x_D = 6 \text{ and } y_D = 4$$

So, $D(6,4)$.



Ex 51: Let $A(-1,2)$, $B(1,5)$, and $C(3,-1)$.
Find the coordinates of the point D such that $\overrightarrow{AB} = \overrightarrow{CD}$.

$$D = (\boxed{5}, \boxed{2})$$

Answer:

- First, compute the vectors:

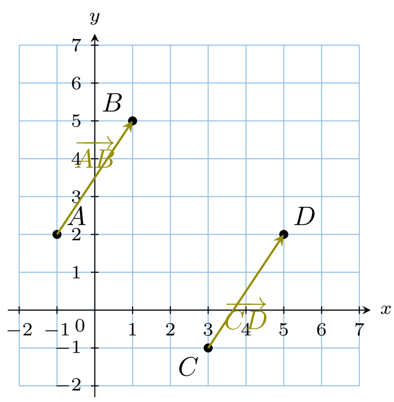
$$\overrightarrow{AB} = \begin{pmatrix} 1-(-1) \\ 5-2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\overrightarrow{CD} = \begin{pmatrix} x_D - 3 \\ y_D - (-1) \end{pmatrix} = \begin{pmatrix} x_D - 3 \\ y_D + 1 \end{pmatrix}$$

- Then, solve the equation:

$$\overrightarrow{AB} = \overrightarrow{CD}$$
$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} x_D - 3 \\ y_D + 1 \end{pmatrix}$$
$$2 = x_D - 3 \text{ and } 3 = y_D + 1$$
$$x_D = 5 \text{ and } y_D = 2$$

So, $D(5,2)$.



D.3 SOLVING VECTOR EQUATIONS

Ex 52: Determine the values of x and y for the following vector equality:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$x = \boxed{2} \text{ and } y = \boxed{3}$$

Answer: For two vectors to be equal, their corresponding components must be equal.
By identifying the components of the two vectors:

$$x = 2$$

$$y = 3$$

Ex 53: Determine the values of x and y for the following vector equality:

$$x\overrightarrow{i} + y\overrightarrow{j} = -2\overrightarrow{i} + \overrightarrow{j}$$

$$x = \boxed{-2} \text{ and } y = \boxed{1}$$

Answer: By identifying the coefficients for each vector of the base, we have

$$x = -2$$

$$y = 1$$

Ex 54: Determine the values of x and y for the following vector equality:

$$\begin{pmatrix} x \\ y+1 \end{pmatrix} = \begin{pmatrix} 2x-1 \\ 3-x \end{pmatrix}$$

$$x = \boxed{1} \text{ and } y = \boxed{1}$$

Answer: For two vectors to be equal, their corresponding components must be equal. By equating the components, we get a system of two linear equations:

$$\begin{cases} x = 2x - 1 \\ y + 1 = 3 - x \end{cases}$$

First, we solve the first equation for x:

$$\begin{aligned} x &= 2x - 1 \\ 1 &= 2x - x \\ x &= 1 \end{aligned}$$

Next, we substitute $x = 1$ into the second equation to find y:

$$\begin{aligned} y + 1 &= 3 - (1) \\ y + 1 &= 2 \\ y &= 2 - 1 \\ y &= 1 \end{aligned}$$

So, the solution is $x = 1$ and $y = 1$.

Ex 55: Determine the values of x and y for the following vector equality:

$$x\overrightarrow{i} + y\overrightarrow{j} = 2y\overrightarrow{i} + 3\overrightarrow{j}$$

$$x = \boxed{6} \text{ and } y = \boxed{3}$$

Answer: We equate the coefficients of the base vectors $\overrightarrow{i}$ and $\overrightarrow{j}$:

$$x\overrightarrow{i} + y\overrightarrow{j} = 2y\overrightarrow{i} + 3\overrightarrow{j}$$

This gives us a system of two linear equations:

$$\begin{cases} x = 2y \\ y = 3 \end{cases}$$

From the second equation, we immediately see that $y = 3$.
We can then substitute this value into the first equation to find x:

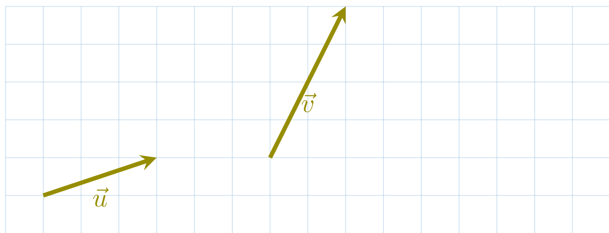
$$\begin{aligned} x &= 2(3) \\ x &= 6 \end{aligned}$$

The solution is $x = 6$ and $y = 3$.

E VECTOR ADDITION AND SUBTRACTION

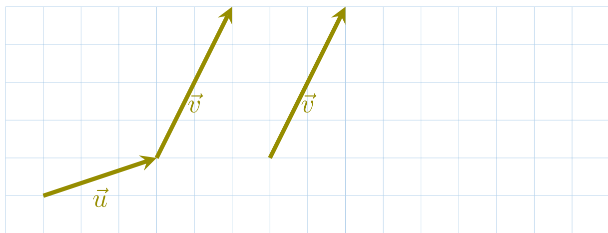
E.1 DRAWING THE SUM OF TWO VECTORS

Ex 56: Draw the arrows diagram of the vector $\vec{u} + \vec{v}$.

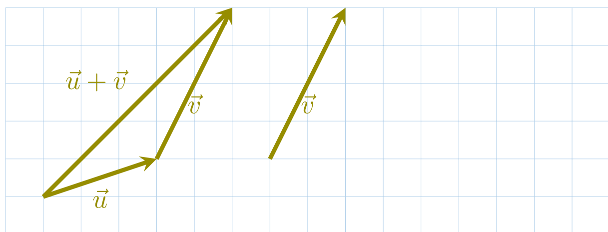


Answer: To add $\vec{u}$ and $\vec{v}$:

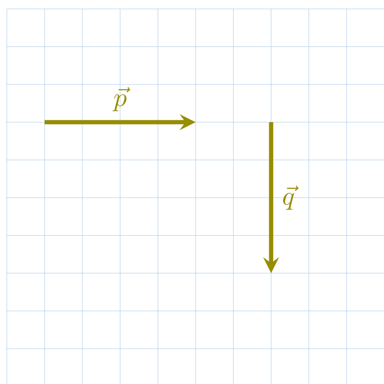
1. At the arrowhead end of $\vec{u}$, draw $\vec{v}$ starting from there (keep the same length and direction).



2. Draw the resulting vector from the start of $\vec{u}$ to the tip of the new $\vec{v}$. This vector is $\vec{u} + \vec{v}$.

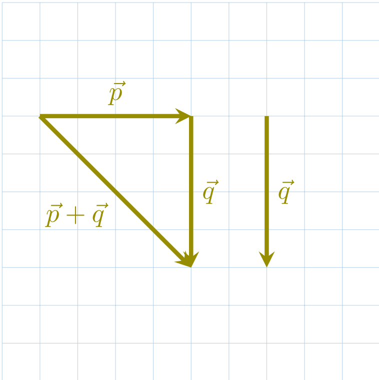


Ex 57: Draw the arrows diagram of the vector $\vec{p} + \vec{q}$.

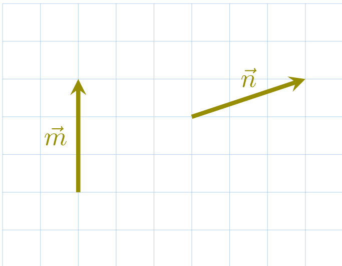


Answer: To add $\vec{p}$ and $\vec{q}$:

1. Place $\vec{q}$ starting at the tip of $\vec{p}$ (preserving its direction and length).
2. Draw the vector from the tail of $\vec{p}$ to the tip of this new $\vec{q}$. This is $\vec{p}+\vec{q}$.

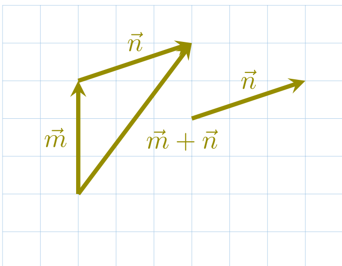


Ex 58: Draw the arrows diagram of the vector $\vec{m}+\vec{n}$.



Answer: To add $\vec{m}$ and $\vec{n}$:

1. Draw $\vec{n}$ starting at the tip of $\vec{m}$ (same direction and length as the original).
2. Draw the resulting vector from the origin of $\vec{m}$ to the tip of this new $\vec{n}$. This is $\vec{m}+\vec{n}$.



E.2 CALCULATING THE SUM OF VECTORS

Ex 59: Calculate the sum of the vectors $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$.

$$\vec{a}+\vec{b} = \begin{pmatrix} \boxed{-3} \\ \boxed{1} \end{pmatrix}$$

Answer:

$$\begin{aligned} \vec{a}+\vec{b} &= \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} 2+(-5) \\ (-3)+4 \end{pmatrix} \\ &= \begin{pmatrix} -3 \\ 1 \end{pmatrix} \end{aligned}$$

Ex 60: Calculate the sum of the vectors $\vec{u} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$.

$$\vec{u}+\vec{v} = \begin{pmatrix} \boxed{3} \\ \boxed{7} \end{pmatrix}$$

Answer:

$$\begin{aligned} \vec{u}+\vec{v} &= \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 5 \end{pmatrix} \\ &= \begin{pmatrix} 4+(-1) \\ 2+5 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ 7 \end{pmatrix} \end{aligned}$$

Ex 61: Calculate the sum of the vectors $\vec{p} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$.

$$\vec{p}+\vec{q} = \begin{pmatrix} \boxed{5} \\ \boxed{2} \end{pmatrix}$$

Answer:

$$\begin{aligned} \vec{p}+\vec{q} &= \begin{pmatrix} -3 \\ 6 \end{pmatrix} + \begin{pmatrix} 8 \\ -4 \end{pmatrix} \\ &= \begin{pmatrix} (-3)+8 \\ 6+(-4) \end{pmatrix} \\ &= \begin{pmatrix} 5 \\ 2 \end{pmatrix} \end{aligned}$$

Ex 62: Calculate the sum of the vectors $\vec{m} = \begin{pmatrix} 0 \\ -7 \end{pmatrix}$ and $\vec{n} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$.

$$\vec{m}+\vec{n} = \begin{pmatrix} \boxed{5} \\ \boxed{-4} \end{pmatrix}$$

Answer:

$$\begin{aligned} \vec{m}+\vec{n} &= \begin{pmatrix} 0 \\ -7 \end{pmatrix} + \begin{pmatrix} 5 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 0+5 \\ (-7)+3 \end{pmatrix} \\ &= \begin{pmatrix} 5 \\ -4 \end{pmatrix} \end{aligned}$$

E.3 RECOGNIZING SUMS OF VECTORS

MCQ 63: Calculate the sum of vectors: $\overrightarrow{AB}+\overrightarrow{BC}$.

- ☐ $\overrightarrow{CA}$
- ☒ $\overrightarrow{AC}$
- ☐ $\overrightarrow{BA}$
- ☐ $\overrightarrow{CB}$

Answer:

$$\overrightarrow{AB}+\overrightarrow{BC} = \overrightarrow{AC} \quad \text{(by Chasles' relation)}$$

MCQ 64: Calculate the sum of vectors: $\overrightarrow{BC}+\overrightarrow{AB}$.

☐ $\overrightarrow{CB}$

☐ $\overrightarrow{BA}$

☐ $\overrightarrow{0}$

☒ $\overrightarrow{AC}$

Answer:

$$\begin{aligned}\overrightarrow{BC}+\overrightarrow{AB} &= \overrightarrow{AB}+\overrightarrow{BC}\\ &= \overrightarrow{AC} \quad (\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC} \text{ by Chasles' relation})\end{aligned}$$

MCQ 65: Calculate the sum of vectors: $\overrightarrow{AB}+\overrightarrow{BA}$.

☐ $\overrightarrow{BA}$

☐ $\overrightarrow{AB}$

☒ $\overrightarrow{0}$

Answer:

$$\begin{aligned}\overrightarrow{AB}+\overrightarrow{BA} &= \overrightarrow{AA}\\ &= \overrightarrow{0}\end{aligned}$$

MCQ 66: Calculate the sum of vectors: $\overrightarrow{EA}+\overrightarrow{AB}+\overrightarrow{BC}$.

☐ $\overrightarrow{CE}$

☐ $\overrightarrow{0}$

☐ $\overrightarrow{AC}$

☒ $\overrightarrow{EC}$

Answer:

$$\begin{aligned}\overrightarrow{EA}+\overrightarrow{AB}+\overrightarrow{BC} &= \overrightarrow{EA}+\overrightarrow{AC}\\ &= \overrightarrow{EC}\end{aligned}$$

MCQ 67: Calculate the sum of vectors: $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}$.

☒ $\overrightarrow{AD}$

☐ $\overrightarrow{DA}$

☐ $\overrightarrow{BD}$

☐ $\overrightarrow{0}$

Answer:

$$\begin{aligned}\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD} &= \overrightarrow{AC}+\overrightarrow{CD}\\ &= \overrightarrow{AD}\end{aligned}$$

E.4 CALCULATING THE SUM OF VECTORS IN 3D

Ex 68: Calculate the sum of the vectors $\vec{a}=\begin{pmatrix}2\\-3\\0\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}-5\\4\\2\end{pmatrix}$.

$$\vec{a}+\vec{b}=\begin{pmatrix}\boxed{-3}\\\boxed{1}\\\boxed{2}\end{pmatrix}$$

Answer:

$$\begin{aligned}\vec{a}+\vec{b} &= \begin{pmatrix}2\\-3\\0\end{pmatrix}+\begin{pmatrix}-5\\4\\2\end{pmatrix}\\ &= \begin{pmatrix}2+(-5)\\(-3)+4\\0+2\end{pmatrix}\\ &= \begin{pmatrix}-3\\1\\2\end{pmatrix}\end{aligned}$$

Ex 69: Calculate the sum of the vectors $\vec{u}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\vec{v}=\begin{pmatrix}4\\5\\6\end{pmatrix}$.

$$\vec{u}+\vec{v}=\begin{pmatrix}\boxed{5}\\\boxed{7}\\\boxed{9}\end{pmatrix}$$

Answer:

$$\begin{aligned}\vec{u}+\vec{v} &= \begin{pmatrix}1\\2\\3\end{pmatrix}+\begin{pmatrix}4\\5\\6\end{pmatrix}\\ &= \begin{pmatrix}1+4\\2+5\\3+6\end{pmatrix}\\ &= \begin{pmatrix}5\\7\\9\end{pmatrix}\end{aligned}$$

Ex 70: Calculate the sum of the vectors $\vec{m}=\begin{pmatrix}-1\\0\\5\end{pmatrix}$ and $\vec{n}=\begin{pmatrix}3\\-2\\-4\end{pmatrix}$.

$$\vec{m}+\vec{n}=\begin{pmatrix}\boxed{2}\\\boxed{-2}\\\boxed{1}\end{pmatrix}$$

Answer:

$$\begin{aligned}\vec{m}+\vec{n} &= \begin{pmatrix}-1\\0\\5\end{pmatrix}+\begin{pmatrix}3\\-2\\-4\end{pmatrix}\\ &= \begin{pmatrix}-1+3\\0+(-2)\\5+(-4)\end{pmatrix}\\ &= \begin{pmatrix}2\\-2\\1\end{pmatrix}\end{aligned}$$

Ex 71: Calculate the sum of the vectors $\vec{p}=\begin{pmatrix}10\\-8\\6\end{pmatrix}$ and $\vec{q}=\begin{pmatrix}-2\\8\\-3\end{pmatrix}$.

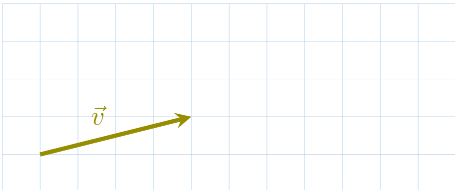
$$\vec{p}+\vec{q}=\begin{pmatrix}\boxed{8}\\\boxed{0}\\\boxed{3}\end{pmatrix}$$

Answer:

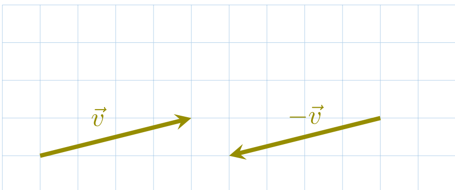
$$\begin{aligned}\vec{p}+\vec{q}&=\begin{pmatrix}10\\-8\\6\end{pmatrix}+\begin{pmatrix}-2\\8\\-3\end{pmatrix}\\&=\begin{pmatrix}10+(-2)\\-8+8\\6+(-3)\end{pmatrix}\\&=\begin{pmatrix}8\\0\\3\end{pmatrix}\end{aligned}$$

E.5 DRAWING THE NEGATIVE OF A VECTOR

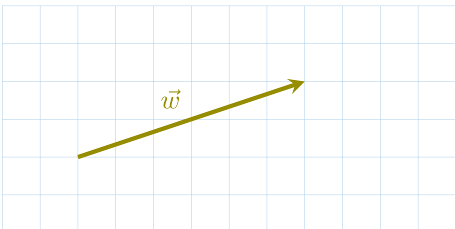
Ex 72: Draw the negative vector of $\vec{v}$.



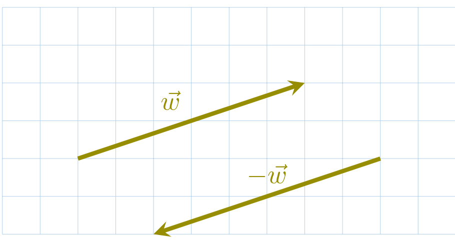
Answer: Draw a vector with the same direction, **opposite** sense, and length as $\vec{v}$, starting from any point on the grid. For example:



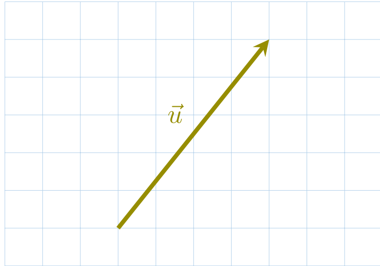
Ex 73: Draw the negative vector of $\vec{w}$.



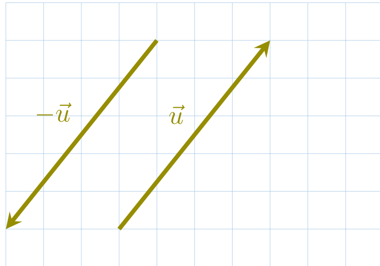
Answer: Draw a vector with the same direction, **opposite** sense, and length as $\vec{w}$, starting from any point on the grid. For example:



Ex 74: Draw the negative vector of $\vec{u}$.



Answer: Draw a vector with the same direction, **opposite** sense, and length as $\vec{u}$, starting from any point on the grid. For example:



E.6 CALCULATING THE NEGATIVE OF A VECTOR

Ex 75: Calculate the negative of the vector $\vec{a}=\begin{pmatrix}4\\-7\end{pmatrix}$.

$$-\vec{a}=\begin{pmatrix}\boxed{-4}\\\boxed{7}\end{pmatrix}$$

Answer:

$$\begin{aligned}-\vec{a}&=-\begin{pmatrix}4\\-7\end{pmatrix}\\&=\begin{pmatrix}-4\\7\end{pmatrix}\end{aligned}$$

Ex 76: Calculate the negative of the vector $\vec{b}=\begin{pmatrix}-3\\5\end{pmatrix}$.

$$-\vec{b}=\begin{pmatrix}\boxed{3}\\\boxed{-5}\end{pmatrix}$$

Answer:

$$\begin{aligned}-\vec{b}&=-\begin{pmatrix}-3\\5\end{pmatrix}\\&=\begin{pmatrix}3\\-5\end{pmatrix}\end{aligned}$$

Ex 77: Calculate the negative of the vector $\vec{u}=\begin{pmatrix}6\\2\end{pmatrix}$.

$$-\vec{u}=\begin{pmatrix}\boxed{-6}\\\boxed{-2}\end{pmatrix}$$

Answer:

$$\begin{aligned}-\vec{u}&=-\begin{pmatrix}6\\2\end{pmatrix}\\&=\begin{pmatrix}-6\\-2\end{pmatrix}\end{aligned}$$

Ex 78: Calculate the negative of the vector $\vec{p}=\begin{pmatrix}0\\-8\end{pmatrix}$.

$$-\vec{p} = \begin{pmatrix} \boxed{0} \\ \boxed{8} \end{pmatrix}$$

Answer:

$$\begin{aligned} -\vec{p} &= -\begin{pmatrix} 0 \\ -8 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 8 \end{pmatrix} \end{aligned}$$

E.7 CALCULATING THE DIFFERENCE OF VECTORS

Ex 79: Calculate the difference of the vectors $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$.

$$\vec{a} - \vec{b} = \begin{pmatrix} \boxed{7} \\ \boxed{-7} \end{pmatrix}$$

Answer:

$$\begin{aligned} \vec{a} - \vec{b} &= \begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} -5 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} 2-(-5) \\ -3-4 \end{pmatrix} \\ &= \begin{pmatrix} 7 \\ -7 \end{pmatrix} \end{aligned}$$

Ex 80: Calculate the difference of the vectors $\vec{u} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$.

$$\vec{u} - \vec{v} = \begin{pmatrix} \boxed{5} \\ \boxed{-3} \end{pmatrix}$$

Answer:

$$\begin{aligned} \vec{u} - \vec{v} &= \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \end{pmatrix} \\ &= \begin{pmatrix} 4-(-1) \\ 2-5 \end{pmatrix} \\ &= \begin{pmatrix} 5 \\ -3 \end{pmatrix} \end{aligned}$$

Ex 81: Calculate the difference of the vectors $\vec{p} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$.

$$\vec{p} - \vec{q} = \begin{pmatrix} \boxed{-11} \\ \boxed{10} \end{pmatrix}$$

Answer:

$$\begin{aligned} \vec{p} - \vec{q} &= \begin{pmatrix} -3 \\ 6 \end{pmatrix} - \begin{pmatrix} 8 \\ -4 \end{pmatrix} \\ &= \begin{pmatrix} -3-8 \\ 6-(-4) \end{pmatrix} \\ &= \begin{pmatrix} -11 \\ 10 \end{pmatrix} \end{aligned}$$

Ex 82: Calculate the difference of the vectors $\vec{m} = \begin{pmatrix} 0 \\ -7 \end{pmatrix}$ and $\vec{n} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$.

$$\vec{m} - \vec{n} = \begin{pmatrix} \boxed{-5} \\ \boxed{-10} \end{pmatrix}$$

Answer:

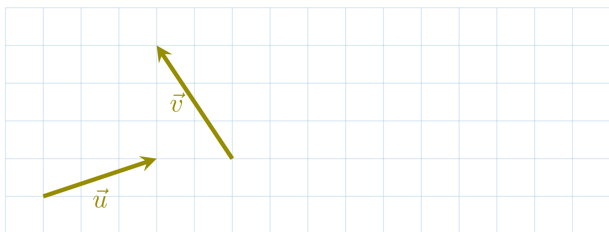
$$\begin{aligned} \vec{m} - \vec{n} &= \begin{pmatrix} 0 \\ -7 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 0-5 \\ -7-3 \end{pmatrix} \\ &= \begin{pmatrix} -5 \\ -10 \end{pmatrix} \end{aligned}$$

E.8 DRAWING THE SUBTRACTION OF TWO VECTORS

Ex 83: Draw the vector of $\vec{u}-\vec{v}$. (Do that on your graph paper.)

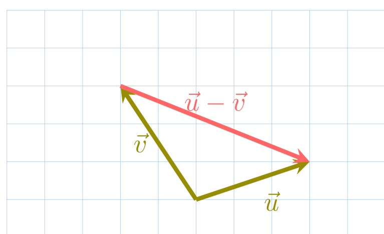
Answer: To graphically subtract two vectors that start at the same origin, we draw the resultant vector from the tip of the second vector ($\vec{v}$) to the tip of the first vector ($\vec{u}$).This vector connects the arrowheads, starting at the one being subtracted.

Ex 84: Draw the vector of $\vec{u}-\vec{v}$. (Do that on your graph paper.)

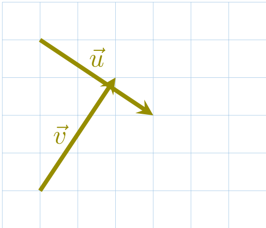


Answer: To graphically subtract $\vec{v}$ from $\vec{u}$, we can move them so they share the same starting point.

1. Choose a common origin on the grid.
2. Translate (move without rotating) both $\vec{u}$ and $\vec{v}$ so their tails are at this common origin.
3. The resultant vector, $\vec{u}-\vec{v}$, is the vector drawn from the tip of the translated $\vec{v}$ to the tip of the translated $\vec{u}$.

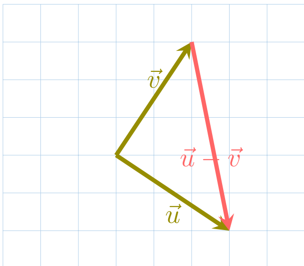


Ex 85: Draw the vector of $\vec{u}-\vec{v}$. (Do that on your graph paper.)



Answer: To graphically subtract $\vec{v}$ from $\vec{u}$, we can move them so they share the same starting point.

1. Choose a common origin on the grid.
2. Translate (move without rotating) both $\vec{u}$ and $\vec{v}$ so their tails are at this common origin.
3. The resultant vector, $\vec{u}-\vec{v}$, is the vector drawn from the tip of the translated $\vec{v}$ to the tip of the translated $\vec{u}$.



E.9 CALCULATING THE DIFFERENCE OF VECTORS IN 3D

Ex 86: Calculate the difference of the vectors $\vec{a}=\begin{pmatrix}1\\5\\-2\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}3\\2\\4\end{pmatrix}$.

$$\vec{a}-\vec{b}=\begin{pmatrix}\boxed{-2}\\\boxed{3}\\\boxed{-6}\end{pmatrix}$$

Answer:

$$\begin{aligned}\vec{a}-\vec{b}&=\begin{pmatrix}1\\5\\-2\end{pmatrix}-\begin{pmatrix}3\\2\\4\end{pmatrix}\\&=\begin{pmatrix}1-3\\5-2\\-2-4\end{pmatrix}\\&=\begin{pmatrix}-2\\3\\-6\end{pmatrix}\end{aligned}$$

Ex 87: Calculate the difference of the vectors $\vec{c}=\begin{pmatrix}-6\\7\\-1\end{pmatrix}$ and $\vec{d}=\begin{pmatrix}-2\\-3\\5\end{pmatrix}$.

$$\vec{c}-\vec{d}=\begin{pmatrix}\boxed{-4}\\\boxed{10}\\\boxed{-6}\end{pmatrix}$$

Answer:

$$\begin{aligned}\vec{c}-\vec{d}&=\begin{pmatrix}-6\\7\\-1\end{pmatrix}-\begin{pmatrix}-2\\-3\\5\end{pmatrix}\\&=\begin{pmatrix}-6-(-2)\\7-(-3)\\-1-5\end{pmatrix}\\&=\begin{pmatrix}-4\\10\\-6\end{pmatrix}\end{aligned}$$

Ex 88: Calculate the difference of the vectors $\vec{e}=\begin{pmatrix}0\\-4\\1\end{pmatrix}$ and $\vec{f}=\begin{pmatrix}5\\0\\-3\end{pmatrix}$.

$$\vec{e}-\vec{f}=\begin{pmatrix}\boxed{-5}\\\boxed{-4}\\\boxed{4}\end{pmatrix}$$

Answer:

$$\begin{aligned}\vec{e}-\vec{f}&=\begin{pmatrix}0\\-4\\1\end{pmatrix}-\begin{pmatrix}5\\0\\-3\end{pmatrix}\\&=\begin{pmatrix}0-5\\-4-0\\1-(-3)\end{pmatrix}\\&=\begin{pmatrix}-5\\-4\\4\end{pmatrix}\end{aligned}$$

F SCALAR MULTIPLICATION

F.1 MULTIPLYING A VECTOR BY A SCALAR

Ex 89: Calculate the product of the vector $\vec{b}=\begin{pmatrix}-5\\4\end{pmatrix}$ by 3.

$$3\vec{b}=\begin{pmatrix}\boxed{-15}\\\boxed{12}\end{pmatrix}$$

Answer:

$$\begin{aligned}3\vec{b}&=3\times\begin{pmatrix}-5\\4\end{pmatrix}\\&=\begin{pmatrix}3\times(-5)\\3\times 4\end{pmatrix}\\&=\begin{pmatrix}-15\\12\end{pmatrix}\end{aligned}$$

Ex 90: Calculate the product of the vector $\vec{u}=\begin{pmatrix}0\\6\end{pmatrix}$ by -2.

$$-2\vec{u}=\begin{pmatrix}\boxed{0}\\\boxed{-12}\end{pmatrix}$$

Answer:

$$\begin{aligned}-2\vec{u}&=-2\times\begin{pmatrix}0\\6\end{pmatrix}\\&=\begin{pmatrix}-2\times 0\\-2\times 6\end{pmatrix}\\&=\begin{pmatrix}0\\-12\end{pmatrix}\end{aligned}$$

Ex 91: Calculate the product of the vector $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ by -4.

$$-4\vec{a} = \begin{pmatrix} \boxed{-8} \\ \boxed{12} \end{pmatrix}$$

Answer:

$$\begin{aligned} -4\vec{a} &= -4 \times \begin{pmatrix} 2 \\ -3 \end{pmatrix} \\ &= \begin{pmatrix} -4 \times 2 \\ -4 \times (-3) \end{pmatrix} \\ &= \begin{pmatrix} -8 \\ 12 \end{pmatrix} \end{aligned}$$

Ex 92: Calculate the product of the vector $\vec{p} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$ by 0.5.

$$\frac{1}{2}\vec{p} = \begin{pmatrix} \boxed{3.5} \\ \boxed{-0.5} \end{pmatrix}$$

Answer:

$$\begin{aligned} 0.5\vec{p} &= 0.5 \times \begin{pmatrix} 7 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} 0.5 \times 7 \\ 0.5 \times (-1) \end{pmatrix} \\ &= \begin{pmatrix} 3.5 \\ -0.5 \end{pmatrix} \end{aligned}$$

F.2 CALCULATING LINEAR COMBINATIONS OF VECTORS

Ex 93: Calculate $3\vec{a} - \vec{b}$ where $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$.

$$3\vec{a} - \vec{b} = \begin{pmatrix} \boxed{11} \\ \boxed{-13} \end{pmatrix}$$

Answer:

$$\begin{aligned} 3\vec{a} - \vec{b} &= 3 \times \begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} -5 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} 3 \times 2 \\ 3 \times (-3) \end{pmatrix} - \begin{pmatrix} -5 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} 6 \\ -9 \end{pmatrix} - \begin{pmatrix} -5 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} 6 - (-5) \\ -9 - 4 \end{pmatrix} \\ &= \begin{pmatrix} 6 + (+5) \\ -9 - 4 \end{pmatrix} \\ &= \begin{pmatrix} 11 \\ -13 \end{pmatrix} \end{aligned}$$

Ex 94: Calculate $2(\vec{u} + 2\vec{v})$ where $\vec{u} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

$$2(\vec{u} + 2\vec{v}) = \begin{pmatrix} \boxed{14} \\ \boxed{16} \end{pmatrix}$$

Answer:

$$\begin{aligned} 2(\vec{u} + 2\vec{v}) &= 2\left(\begin{pmatrix} 1 \\ -2 \end{pmatrix} + 2 \times \begin{pmatrix} 3 \\ 5 \end{pmatrix}\right) \\ &= 2\left(\begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 6 \\ 10 \end{pmatrix}\right) \\ &= 2\begin{pmatrix} 1 + 6 \\ -2 + 10 \end{pmatrix} \\ &= 2\begin{pmatrix} 7 \\ 8 \end{pmatrix} \\ &= \begin{pmatrix} 14 \\ 16 \end{pmatrix} \end{aligned}$$

Ex 95: Calculate $4\vec{p} - 2\vec{q}$ where $\vec{p} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

$$4\vec{p} - 2\vec{q} = \begin{pmatrix} \boxed{-8} \\ \boxed{22} \end{pmatrix}$$

Answer:

$$\begin{aligned} 4\vec{p} - 2\vec{q} &= 4\begin{pmatrix} -1 \\ 3 \end{pmatrix} - 2\begin{pmatrix} 2 \\ -5 \end{pmatrix} \\ &= \begin{pmatrix} 4 \times -1 \\ 4 \times 3 \end{pmatrix} - \begin{pmatrix} 2 \times 2 \\ 2 \times -5 \end{pmatrix} \\ &= \begin{pmatrix} -4 \\ 12 \end{pmatrix} - \begin{pmatrix} 4 \\ -10 \end{pmatrix} \\ &= \begin{pmatrix} -4 - 4 \\ 12 - (-10) \end{pmatrix} \\ &= \begin{pmatrix} -8 \\ 22 \end{pmatrix} \end{aligned}$$

Ex 96: Calculate $-3\vec{u} + 5\vec{v}$ where $\vec{u} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$.

$$-3\vec{u} + 5\vec{v} = \begin{pmatrix} \boxed{-11} \\ \boxed{20} \end{pmatrix}$$

Answer:

$$\begin{aligned} -3\vec{u} + 5\vec{v} &= -3\begin{pmatrix} 2 \\ 0 \end{pmatrix} + 5\begin{pmatrix} -1 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} -6 \\ 0 \end{pmatrix} + \begin{pmatrix} -5 \\ 20 \end{pmatrix} \\ &= \begin{pmatrix} -6 + (-5) \\ 0 + 20 \end{pmatrix} \\ &= \begin{pmatrix} -11 \\ 20 \end{pmatrix} \end{aligned}$$

F.3 DETERMINING THE IMAGE OF A POINT UNDER A HOMOTHETY

Ex 97: Let $O(0,0)$ and $M(3,-2)$. The point M' is the image of M by the homothety of center O and ratio $k = 2$ so that $2\overrightarrow{OM} = \overrightarrow{OM'}$.
Find the coordinates of M'.

$$M' = (\boxed{6}, \boxed{-4})$$

Answer:

- $$\begin{aligned} \overrightarrow{OM'} &= 2\overrightarrow{OM} \\ &= 2\begin{pmatrix} 3 \\ -2 \end{pmatrix} \\ &= \begin{pmatrix} 6 \\ -4 \end{pmatrix} \end{aligned}$$

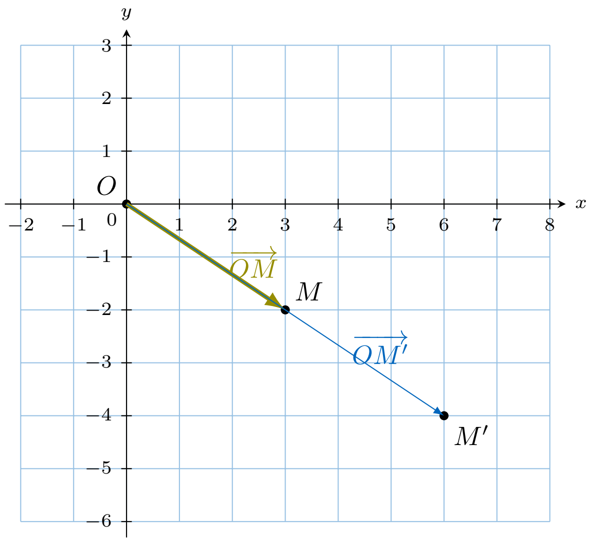
- $\overrightarrow{OM'} = \begin{pmatrix} x_{M'} - x_O \\ y_{M'} - y_O \end{pmatrix}$

  $\begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} x_{M'} - 0 \\ y_{M'} - 0 \end{pmatrix}$

  By identification,

  - $x_{M'} - 0 = 6$, hence $x_{M'} = 6$.
  - $y_{M'} - 0 = -4$, hence $y_{M'} = -4$.

So $M'(6, -4)$.



Ex 98: Let $A(2, -1)$ and $M(3, 1)$. The point M' is the image of M by the homothety of center A and ratio $k = -2$ so that $\overrightarrow{AM'} = -2\,\overrightarrow{AM}$.
Find the coordinates of M'.

$$M' = (\boxed{0}, \boxed{-5})$$

Answer:

- $\overrightarrow{AM} = \begin{pmatrix} 3-2 \\ 1-(-1) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

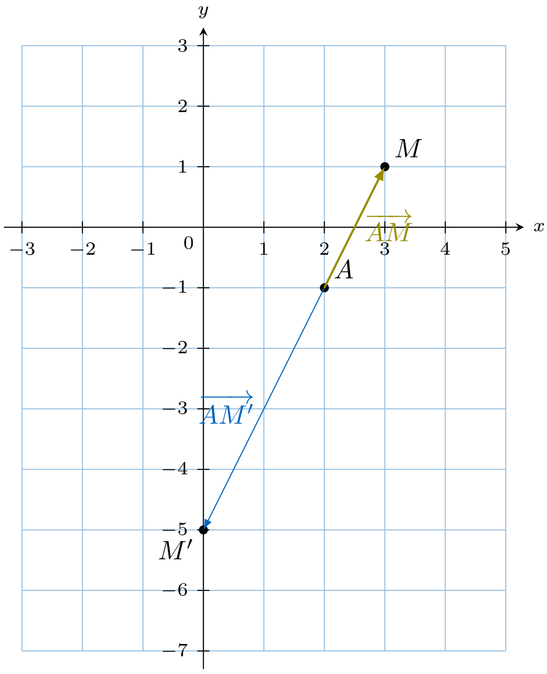
- $\overrightarrow{AM'} = -2\,\overrightarrow{AM} = -2\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

- $\overrightarrow{AM'} = \begin{pmatrix} x_{M'} - x_A \\ y_{M'} - y_A \end{pmatrix}$ By identification,

  $\begin{pmatrix} -2 \\ -4 \end{pmatrix} = \begin{pmatrix} x_{M'} - 2 \\ y_{M'} - (-1) \end{pmatrix}$

  - $x_{M'} - 2 = -2 \implies x_{M'} = 0$
  - $y_{M'} - (-1) = -4 \implies y_{M'} = -5$

So $M'(0, -5)$.



Ex 99: Let $A(2, -1)$ and $M(3, 1)$. The point M' is the image of M by the homothety of center A and ratio $k = 3$, so that $\overrightarrow{AM'} = 3\,\overrightarrow{AM}$.
Find the coordinates of M'.

$$M' = (\boxed{5}, \boxed{5})$$

Answer:

- $\overrightarrow{AM} = \begin{pmatrix} 3-2 \\ 1-(-1) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

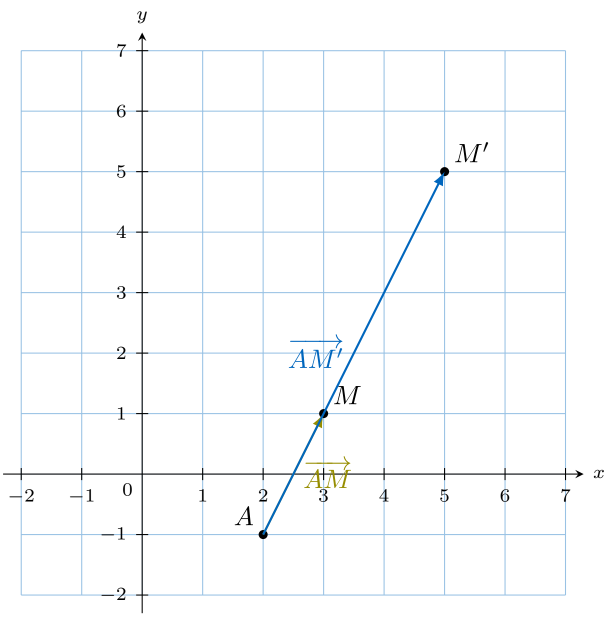
- $\overrightarrow{AM'} = 3\,\overrightarrow{AM} = 3\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

- $\overrightarrow{AM'} = \begin{pmatrix} x_{M'} - x_A \\ y_{M'} - y_A \end{pmatrix}$ By identification,

  $\begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} x_{M'} - 2 \\ y_{M'} - (-1) \end{pmatrix}$

  - $x_{M'} - 2 = 3 \implies x_{M'} = 5$
  - $y_{M'} - (-1) = 6 \implies y_{M'} = 5$

So $M'(5, 5)$.



F.4 CALCULATING LINEAR COMBINATIONS OF VECTORS IN 3D

Ex 100: Calculate $3\vec{a}-\vec{b}$ where $\vec{a}=\begin{pmatrix}2\\-3\\1\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}-5\\4\\-2\end{pmatrix}$.

$$3\vec{a}-\vec{b}=\begin{pmatrix}\boxed{11}\\\boxed{-13}\\\boxed{5}\end{pmatrix}$$

Answer:

$$\begin{aligned}3\vec{a}-\vec{b}&=3\begin{pmatrix}2\\-3\\1\end{pmatrix}-\begin{pmatrix}-5\\4\\-2\end{pmatrix}\\&=\begin{pmatrix}3\times2\\3\times(-3)\\3\times1\end{pmatrix}-\begin{pmatrix}-5\\4\\-2\end{pmatrix}\\&=\begin{pmatrix}6\\-9\\3\end{pmatrix}-\begin{pmatrix}-5\\4\\-2\end{pmatrix}\\&=\begin{pmatrix}6-(-5)\\-9-4\\3-(-2)\end{pmatrix}\\&=\begin{pmatrix}11\\-13\\5\end{pmatrix}\end{aligned}$$

Ex 101: Calculate $2\vec{u}+4\vec{v}$ where $\vec{u}=\begin{pmatrix}1\\0\\-5\end{pmatrix}$ and $\vec{v}=\begin{pmatrix}3\\-2\\2\end{pmatrix}$.

$$2\vec{u}+4\vec{v}=\begin{pmatrix}\boxed{14}\\\boxed{-8}\\\boxed{-2}\end{pmatrix}$$

Answer:

$$\begin{aligned}2\vec{u}+4\vec{v}&=2\begin{pmatrix}1\\0\\-5\end{pmatrix}+4\begin{pmatrix}3\\-2\\2\end{pmatrix}\\&=\begin{pmatrix}2\times1\\2\times0\\2\times(-5)\end{pmatrix}+\begin{pmatrix}4\times3\\4\times(-2)\\4\times2\end{pmatrix}\\&=\begin{pmatrix}2\\0\\-10\end{pmatrix}+\begin{pmatrix}12\\-8\\8\end{pmatrix}\\&=\begin{pmatrix}2+12\\0+(-8)\\-10+8\end{pmatrix}\\&=\begin{pmatrix}14\\-8\\-2\end{pmatrix}\end{aligned}$$

Ex 102: Calculate $5\vec{p}-2\vec{q}$ where $\vec{p}=\begin{pmatrix}-1\\2\\-2\end{pmatrix}$ and $\vec{q}=\begin{pmatrix}4\\-3\\0\end{pmatrix}$.

$$5\vec{p}-2\vec{q}=\begin{pmatrix}\boxed{-13}\\\boxed{16}\\\boxed{-10}\end{pmatrix}$$

Answer:

$$\begin{aligned}5\vec{p}-2\vec{q}&=5\begin{pmatrix}-1\\2\\-2\end{pmatrix}-2\begin{pmatrix}4\\-3\\0\end{pmatrix}\\&=\begin{pmatrix}5\times(-1)\\5\times2\\5\times(-2)\end{pmatrix}-\begin{pmatrix}2\times4\\2\times(-3)\\2\times0\end{pmatrix}\\&=\begin{pmatrix}-5\\10\\-10\end{pmatrix}-\begin{pmatrix}8\\-6\\0\end{pmatrix}\\&=\begin{pmatrix}-5-8\\10-(-6)\\-10-0\end{pmatrix}\\&=\begin{pmatrix}-13\\16\\-10\end{pmatrix}\end{aligned}$$

G MAGNITUDE AND UNIT VECTORS

G.1 CALCULATING THE LENGTH OF A VECTOR

Ex 103: Calculate the length of $\overrightarrow{v}=\begin{pmatrix}2\\-1\end{pmatrix}$

$$\|\overrightarrow{v}\|=\boxed{\sqrt{5}}\text{ units}$$

Answer:

$$\begin{aligned}\|\overrightarrow{v}\|&=\sqrt{2^2+(-1)^2}\\&=\sqrt{4+1}\\&=\sqrt{5}\text{ units}\end{aligned}$$

Ex 104: Calculate the length of $\overrightarrow{p}=\begin{pmatrix}0\\-5\end{pmatrix}$

$$\|\overrightarrow{p}\|=\boxed{5}\text{ units}$$

Answer:

$$\begin{aligned}\|\overrightarrow{p}\|&=\sqrt{0^2+(-5)^2}\\&=\sqrt{0+25}\\&=\sqrt{25}\\&=5\text{ units}\end{aligned}$$

Ex 105: Calculate the length of $\overrightarrow{u}=\begin{pmatrix}-6\\2\end{pmatrix}$

$$\|\overrightarrow{u}\|=\boxed{\sqrt{40}}\text{ units}$$

Answer:

$$\begin{aligned}\|\overrightarrow{u}\|&=\sqrt{(-6)^2+2^2}\\&=\sqrt{36+4}\\&=\sqrt{40}\text{ units}\end{aligned}$$

Ex 106: Calculate the length of $\overrightarrow{q}=\begin{pmatrix}1\\-1\end{pmatrix}$

$$\|\overrightarrow{q}\|=\boxed{\sqrt{2}}\text{ units}$$

Answer:

$$\begin{aligned}\|\overrightarrow{q}\|&=\sqrt{1^2+(-1)^2}\\&=\sqrt{1+1}\\&=\sqrt{2}\text{ units}\end{aligned}$$

G.2 CALCULATING THE DISTANCE BETWEEN TWO POINTS

Ex 107: Let $A(2,3)$ and $B(7,-1)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{5} \\ \boxed{-4} \end{pmatrix}$$

2. Calculate the distance AB.

$$AB = \boxed{\sqrt{41}} \text{ units}$$

Answer:

1. $\overrightarrow{AB} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \end{pmatrix}$
$= \begin{pmatrix} 7-2 \\ -1-3 \end{pmatrix}$
$= \begin{pmatrix} 5 \\ -4 \end{pmatrix}$

2. $AB = \|\overrightarrow{AB}\|$
$= \sqrt{5^2 + (-4)^2}$
$= \sqrt{25+16}$
$= \sqrt{41}$

Ex 108: Let $A(-2,5)$ and $B(4,2)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{6} \\ \boxed{-3} \end{pmatrix}$$

2. Calculate the distance AB.

$$AB = \boxed{\sqrt{45}} \text{ units}$$

Answer:

1. $\overrightarrow{AB} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \end{pmatrix}$
$= \begin{pmatrix} 4-(-2) \\ 2-5 \end{pmatrix}$
$= \begin{pmatrix} 6 \\ -3 \end{pmatrix}$

2. $AB = \|\overrightarrow{AB}\|$
$= \sqrt{6^2 + (-3)^2}$
$= \sqrt{36+9}$
$= \sqrt{45}$

Ex 109: Let $A(0,-2)$ and $B(-3,6)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{-3} \\ \boxed{8} \end{pmatrix}$$

2. Calculate the distance AB.

$$AB = \boxed{\sqrt{73}} \text{ units}$$

Answer:

1. $\overrightarrow{AB} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \end{pmatrix}$
$= \begin{pmatrix} -3-0 \\ 6-(-2) \end{pmatrix}$
$= \begin{pmatrix} -3 \\ 8 \end{pmatrix}$

2. $AB = \|\overrightarrow{AB}\|$
$= \sqrt{(-3)^2 + 8^2}$
$= \sqrt{9+64}$
$= \sqrt{73}$

G.3 USING COORDINATES TO DETERMINE TRIANGLE TYPES

Ex 110: Let $A(0,0)$, $B(6,0)$, and $C(6,8)$.

1. Calculate the lengths AB, BC, and CA.
   - $AB = \boxed{6}$
   - $BC = \boxed{8}$
   - $CA = \boxed{10}$

2. Calculate the perimeter of triangle ABC.

$$\text{Perimeter} = \boxed{24} \text{ units}$$

Answer:

1.
   - $AB = \sqrt{(6-0)^2 + (0-0)^2} = \sqrt{36} = 6$
   - $BC = \sqrt{(6-6)^2 + (8-0)^2} = \sqrt{64} = 8$
   - $CA = \sqrt{(0-6)^2 + (0-8)^2} = \sqrt{36+64} = \sqrt{100} = 10$

2. Perimeter $= 6 + 8 + 10 = 24$ units

Ex 111: Let $A(0,0)$, $B(4,0)$, and $C(2,4)$.

1. Calculate the lengths AB, BC, and CA.
   - $AB = \boxed{4}$
   - $BC = \boxed{\sqrt{20}}$
   - $CA = \boxed{\sqrt{20}}$

2. Is the triangle ABC isosceles?

Yes

Answer:

1.
   - $AB = \sqrt{(4-0)^2 + (0-0)^2} = \sqrt{16} = 4$
   - $BC = \sqrt{(2-4)^2 + (4-0)^2} = \sqrt{4+16} = \sqrt{20}$

- $CA = \sqrt{(0-2)^2 + (0-4)^2} = \sqrt{4+16} = \sqrt{20}$

2. Yes, the triangle is isosceles because $BC = CA = \sqrt{20}$, so two sides are equal.

Ex 112: Let $A(0,0)$, $B(2,2\sqrt{3})$, and $C(4,0)$.

1. Calculate the lengths AB, BC, and CA.
   - $AB = \boxed{4}$
   - $BC = \boxed{4}$
   - $CA = \boxed{4}$
2. Is the triangle ABC equilateral?

$$\boxed{\textbf{Yes}}$$

Answer:

1. - $AB = \sqrt{(2-0)^2 + (2\sqrt{3}-0)^2} = \sqrt{4+12} = \sqrt{16} = 4$
   - $BC = \sqrt{(4-2)^2 + (0-2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$
   - $CA = \sqrt{(0-4)^2 + (0-0)^2} = \sqrt{16} = 4$
2. Yes, the triangle is equilateral because $AB = BC = CA = 4$.

G.4 CALCULATING THE LENGTH OF A VECTOR IN 3D

Ex 113: Calculate the length of $\overrightarrow{v} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$

$$\|\overrightarrow{v}\| = \boxed{7} \text{ units}$$

Answer:

$$\begin{aligned} \|\overrightarrow{v}\| &= \sqrt{2^2 + 3^2 + 6^2} \\ &= \sqrt{4+9+36} \\ &= \sqrt{49} \\ &= 7 \text{ units} \end{aligned}$$

Ex 114: Calculate the length of $\overrightarrow{u} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$

$$\|\overrightarrow{u}\| = \boxed{5} \text{ units}$$

Answer:

$$\begin{aligned} \|\overrightarrow{u}\| &= \sqrt{4^2 + 0^2 + (-3)^2} \\ &= \sqrt{16+0+9} \\ &= \sqrt{25} \\ &= 5 \text{ units} \end{aligned}$$

Ex 115: Calculate the length of $\overrightarrow{w} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$

$$\|\overrightarrow{w}\| = \boxed{\sqrt{30}} \text{ units}$$

Answer:

$$\begin{aligned} \|\overrightarrow{w}\| &= \sqrt{1^2 + (-2)^2 + 5^2} \\ &= \sqrt{1+4+25} \\ &= \sqrt{30} \text{ units} \end{aligned}$$

G.5 NORMALIZING A VECTOR

Ex 116: Normalize the vector $\overrightarrow{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

Answer: The normalized vector is:

$$\begin{aligned} \frac{\overrightarrow{v}}{\|\overrightarrow{v}\|} &= \frac{\begin{pmatrix} 1 \\ -1 \end{pmatrix}}{\left\|\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\|} \\ &= \frac{\begin{pmatrix} 1 \\ -1 \end{pmatrix}}{\sqrt{(1)^2 + (-1)^2}} \\ &= \frac{\begin{pmatrix} 1 \\ -1 \end{pmatrix}}{\sqrt{2}} \\ &= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} \end{aligned}$$

Ex 117: Normalize the vector $\overrightarrow{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

Answer: The normalized vector is:

$$\begin{aligned} \frac{\overrightarrow{u}}{\|\overrightarrow{u}\|} &= \frac{\begin{pmatrix} 3 \\ 4 \end{pmatrix}}{\left\|\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right\|} \\ &= \frac{\begin{pmatrix} 3 \\ 4 \end{pmatrix}}{\sqrt{(3)^2 + (4)^2}} \\ &= \frac{\begin{pmatrix} 3 \\ 4 \end{pmatrix}}{\sqrt{9+16}} \\ &= \frac{\begin{pmatrix} 3 \\ 4 \end{pmatrix}}{\sqrt{25}} \\ &= \frac{\begin{pmatrix} 3 \\ 4 \end{pmatrix}}{5} \\ &= \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix} \end{aligned}$$

Ex 118: Normalize the vector $\overrightarrow{w} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$.

Answer: The normalized vector is:

$$\frac{\overrightarrow{w}}{\|\overrightarrow{w}\|} = \frac{\begin{pmatrix}-5\\2\end{pmatrix}}{\left\|\begin{pmatrix}-5\\2\end{pmatrix}\right\|} = \frac{\begin{pmatrix}-5\\2\end{pmatrix}}{\sqrt{(-5)^2+(2)^2}} = \frac{\begin{pmatrix}-5\\2\end{pmatrix}}{\sqrt{25+4}} = \frac{\begin{pmatrix}-5\\2\end{pmatrix}}{\sqrt{29}} = \begin{pmatrix}\frac{-5}{\sqrt{29}}\\\frac{2}{\sqrt{29}}\end{pmatrix}$$

Ex 119: Normalize the vector $\overrightarrow{p} = \begin{pmatrix}0\\-7\end{pmatrix}$.

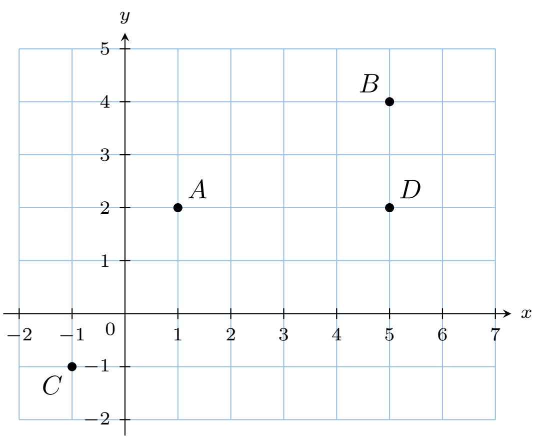
Answer: The normalized vector is:

$$\frac{\overrightarrow{p}}{\|\overrightarrow{p}\|} = \frac{\begin{pmatrix}0\\-7\end{pmatrix}}{\left\|\begin{pmatrix}0\\-7\end{pmatrix}\right\|} = \frac{\begin{pmatrix}0\\-7\end{pmatrix}}{\sqrt{(0)^2+(-7)^2}} = \frac{\begin{pmatrix}0\\-7\end{pmatrix}}{\sqrt{0+49}} = \frac{\begin{pmatrix}0\\-7\end{pmatrix}}{7} = \begin{pmatrix}0\\-1\end{pmatrix}$$

H PARALLEL VECTORS

H.1 TESTING PARALLELISM/ALIGNMENT USING VECTORS

Ex 120:



Let $A(1,2)$, $B(5,4)$, $C(-1,-1)$, and $D(5,2)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix}\boxed{4}\\\boxed{2}\end{pmatrix}$$

2. Calculate the vector $\overrightarrow{CD}$.

$$\overrightarrow{CD} = \begin{pmatrix}\boxed{6}\\\boxed{3}\end{pmatrix}$$

3. Calculate the determinant $\det(\overrightarrow{AB}, \overrightarrow{CD})$.

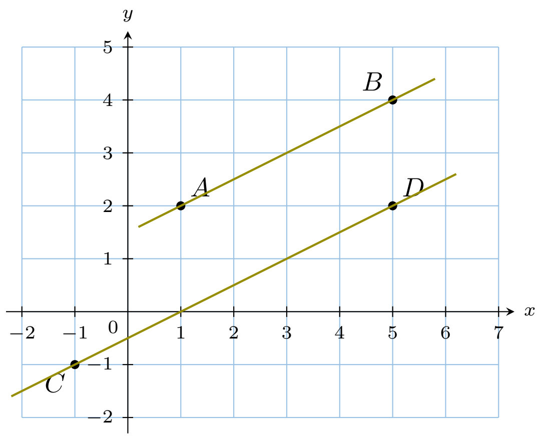
$$\det(\overrightarrow{AB}, \overrightarrow{CD}) = \boxed{0}$$

4. Are the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ parallel?

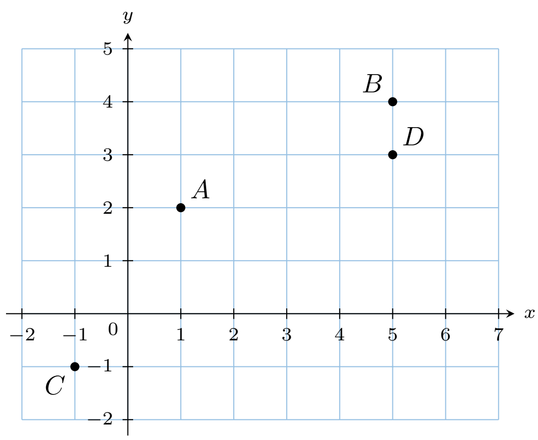
$\boxed{\textbf{Yes}}$

Answer:

1. $\overrightarrow{AB} = \begin{pmatrix}5-1\\4-2\end{pmatrix} = \begin{pmatrix}4\\2\end{pmatrix}$
2. $\overrightarrow{CD} = \begin{pmatrix}5-(-1)\\2-(-1)\end{pmatrix} = \begin{pmatrix}6\\3\end{pmatrix}$
3. $\det(\overrightarrow{AB}, \overrightarrow{CD}) = \begin{vmatrix}4 & 6\\2 & 3\end{vmatrix} = 4\times 3 - 6\times 2 = 12 - 12 = 0$
4. Since the determinant is zero, the vectors are collinear, so the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are **parallel**.



Ex 121:



Let $A(1,2)$, $B(5,4)$, $C(-1,-1)$, and $D(5,3)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{4} \\ \boxed{2} \end{pmatrix}$$

2. Calculate the vector $\overrightarrow{CD}$.

$$\overrightarrow{CD} = \begin{pmatrix} \boxed{6} \\ \boxed{4} \end{pmatrix}$$

3. Calculate the determinant $\det(\overrightarrow{AB}, \overrightarrow{CD})$.

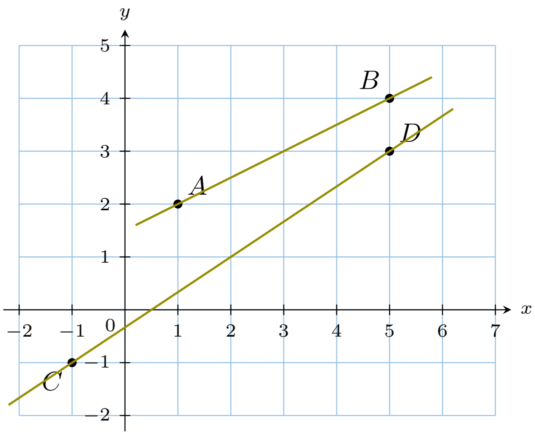
$$\det(\overrightarrow{AB}, \overrightarrow{CD}) = \boxed{4}$$

4. Are the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ parallel?

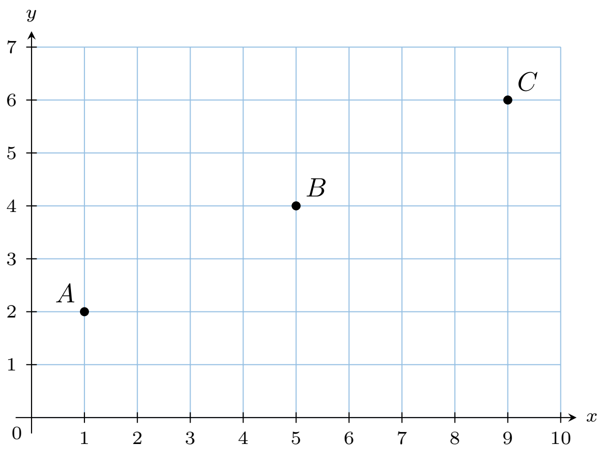
$$\boxed{\textbf{No}}$$

Answer:

1. $\overrightarrow{AB} = \begin{pmatrix} 5-1 \\ 4-2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$
2. $\overrightarrow{CD} = \begin{pmatrix} 5-(-1) \\ 3-(-1) \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$
3. $\det(\overrightarrow{AB}, \overrightarrow{CD}) = \begin{vmatrix} 4 & 6 \\ 2 & 4 \end{vmatrix} = 4 \times 4 - 6 \times 2 = 16 - 12 = 4$
4. The determinant is not zero, so the vectors are not collinear. Therefore, the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are **not parallel**.



Ex 122:



Let $A(1,2)$, $B(5,4)$, and $C(9,6)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{4} \\ \boxed{2} \end{pmatrix}$$

2. Calculate the vector $\overrightarrow{AC}$.

$$\overrightarrow{AC} = \begin{pmatrix} \boxed{8} \\ \boxed{4} \end{pmatrix}$$

3. Calculate the determinant $\det(\overrightarrow{AB}, \overrightarrow{AC})$.

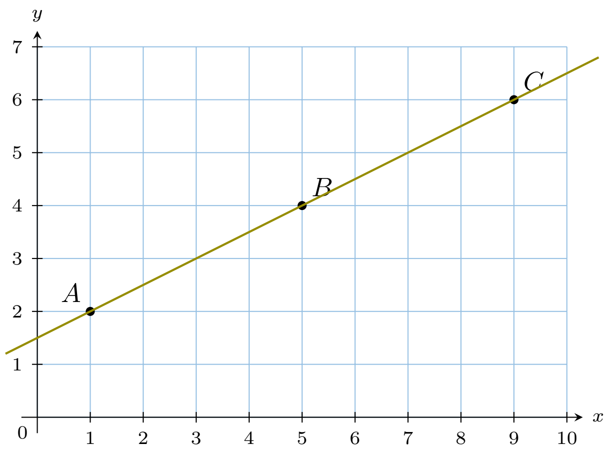
$$\det(\overrightarrow{AB}, \overrightarrow{AC}) = \boxed{0}$$

4. Are the points A, B, and C aligned?

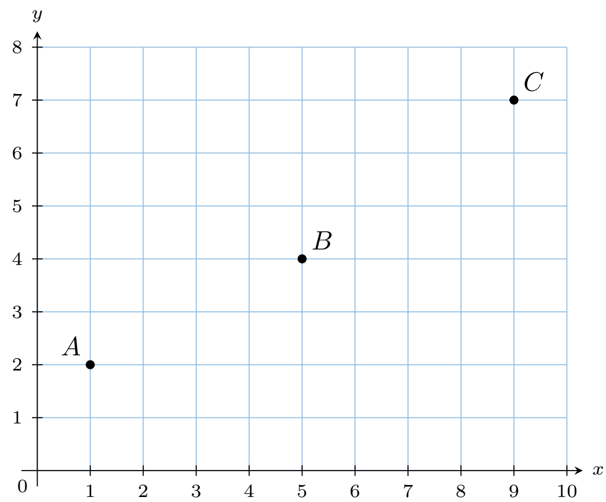
$$\boxed{\textbf{Yes}}$$

Answer:

1. $\overrightarrow{AB} = \begin{pmatrix} 5-1 \\ 4-2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$
2. $\overrightarrow{AC} = \begin{pmatrix} 9-1 \\ 6-2 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$
3. $\det(\overrightarrow{AB}, \overrightarrow{AC}) = \begin{vmatrix} 4 & 8 \\ 2 & 4 \end{vmatrix} = 4 \times 4 - 8 \times 2 = 16 - 16 = 0$
4. Since the determinant is zero, the vectors are collinear. Therefore, the points A, B, and C are aligned.



Ex 123:



Let $A(1,2)$, $B(5,4)$, and $C(9,7)$.

1. Calculate the vector $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \begin{pmatrix} \boxed{4} \\ \boxed{2} \end{pmatrix}$$

2. Calculate the vector $\overrightarrow{AC}$.

$$\overrightarrow{AC} = \begin{pmatrix} \boxed{8} \\ \boxed{5} \end{pmatrix}$$

3. Calculate the determinant $\det(\overrightarrow{AB}, \overrightarrow{AC})$.

$$\det(\overrightarrow{AB}, \overrightarrow{AC}) = \boxed{4}$$

4. Are the points A, B, and C aligned?

$$\boxed{\textbf{No}}$$

Answer:

1. $\overrightarrow{AB} = \begin{pmatrix} 5-1 \\ 4-2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$
2. $\overrightarrow{AC} = \begin{pmatrix} 9-1 \\ 7-2 \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \end{pmatrix}$
3. $\det(\overrightarrow{AB}, \overrightarrow{AC}) = \begin{vmatrix} 4 & 8 \\ 2 & 5 \end{vmatrix} = 4 \times 5 - 8 \times 2 = 20 - 16 = 4$
4. Since the determinant is not zero, the vectors are not collinear. Therefore, the points A, B, and C are **not aligned**.