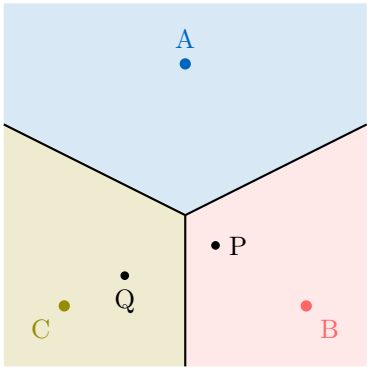
VORONOI DIAGRAMS

A DEFINITIONS

A.1 READING VORONOI DIAGRAMS

Ex 1: Consider the Voronoi diagram below for sites A, B, and C.



1. How many Voronoi regions are there?

   ☐

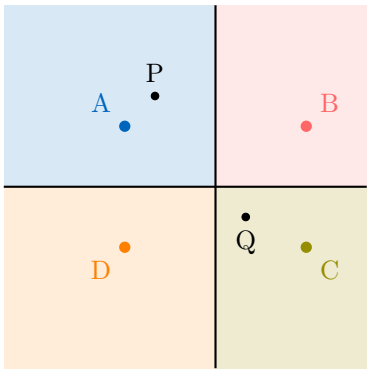
2. Which site is closest to point P?

   ☐ A
   ☐ B
   ☐ C

3. Which site is closest to point Q?

   ☐ A
   ☐ B
   ☐ C

Ex 2: Consider the Voronoi diagram below for sites A, B, C, and D.



1. How many Voronoi regions are there?

   ☐

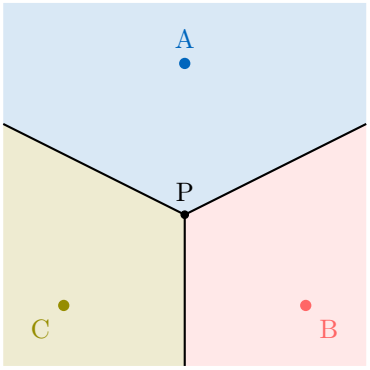
2. Which site is closest to point P?

   ☐ A
   ☐ B
   ☐ C
   ☐ D

3. Which site is closest to point Q?

   ☐ A
   ☐ B
   ☐ C
   ☐ D

MCQ 3:

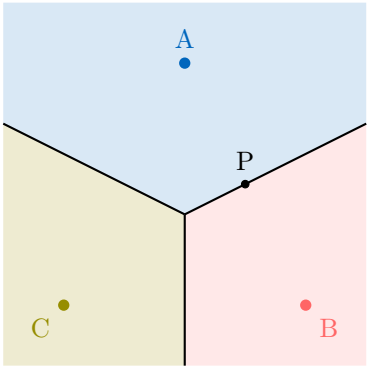


The point P is equidistant from the sites A, B, and C.

☐ True

☐ False

MCQ 4:



The point P is equidistant from sites A and B.

☐ True

☐ False

MCQ 5: In a Voronoi diagram, what is true about a point lying exactly on an edge?

☐ It is closest to all sites in the diagram.

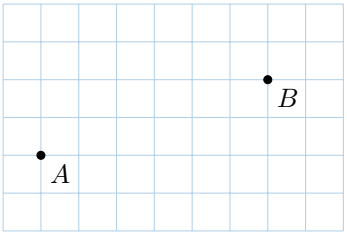
☐ It is equidistant from the two sites sharing that edge.

☐ It is the location of a site.

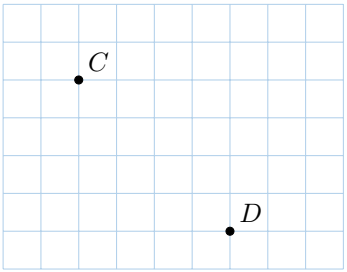
B CONSTRUCTING A VORONOI DIAGRAM

B.1 CONSTRUCTING VORONOI DIAGRAM FOR 2 SITES

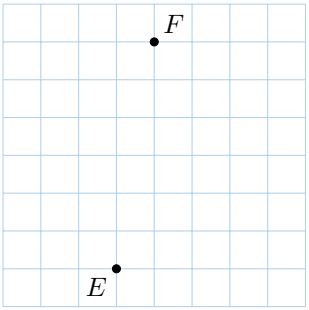
Ex 6: Using a ruler and a set square, draw the Voronoi diagram for two sites A and B.



Ex 7: Using a ruler and a set square, draw the Voronoi diagram for two sites C and D.



Ex 8: Using a ruler and a set square, draw the Voronoi diagram for two sites $E(3,1)$ and $F(4,7)$.



B.2 FINDING THE EQUATION OF THE PERPENDICULAR BISECTOR

Ex 9: Consider two sites $A(0,0)$ and $B(4,2)$.

1. Calculate the coordinates of the midpoint M of the segment $[AB]$.
2. Calculate the gradient (slope) of the segment $[AB]$.
3. Determine the gradient of the perpendicular bisector.
4. Find the equation of the perpendicular bisector in the form $y = mx + c$.

Ex 10: Consider two sites $C(-2,4)$ and $D(2,-2)$.

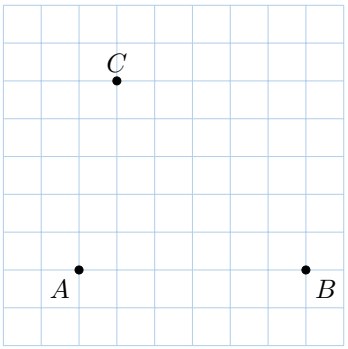
1. Calculate the coordinates of the midpoint M of the segment $[CD]$.
2. Calculate the gradient (slope) of the segment $[CD]$.
3. Determine the gradient of the perpendicular bisector.
4. Find the equation of the perpendicular bisector in the form $y = mx + c$.

Ex 11: Consider two sites $E(-3,-1)$ and $F(1,3)$.

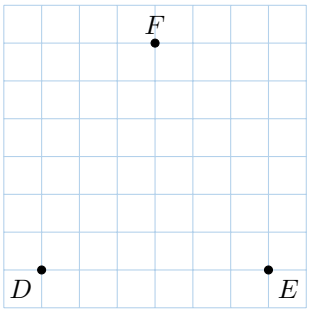
1. Calculate the coordinates of the midpoint M of the segment $[EF]$.
2. Calculate the gradient (slope) of the segment $[EF]$.
3. Determine the gradient of the perpendicular bisector.
4. Find the equation of the perpendicular bisector in the form $y = mx + c$.

B.3 CONSTRUCTING VORONOI DIAGRAM FOR 3 SITES

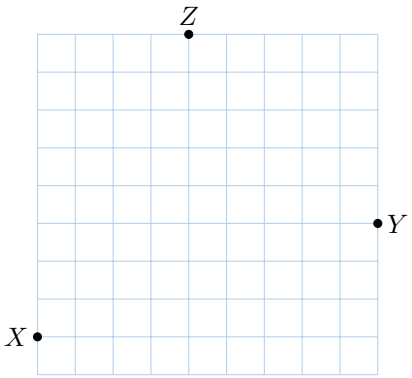
Ex 12: Using a ruler and a set square, draw the Voronoi diagram for three sites A, B, and C.



Ex 13: Using a ruler and a set square, draw the Voronoi diagram for three sites D, E, and F.



Ex 14: Using a ruler and a set square, draw the Voronoi diagram for three sites X, Y, and Z.



B.4 CONSTRUCTING VORONOI DIAGRAM WITH COORDINATES

Ex 15: Consider three sites $A(0,0)$, $B(6,0)$, and $C(2,4)$.
The perpendicular bisector of $[AB]$ has equation $x = 3$.

1. Find the equation of the perpendicular bisector of $[AC]$.
2. Find the coordinates of the Voronoi vertex V where the edges meet.

Ex 16: Consider three sites $X(0,1)$, $Y(9,4)$, and $Z(4,9)$.

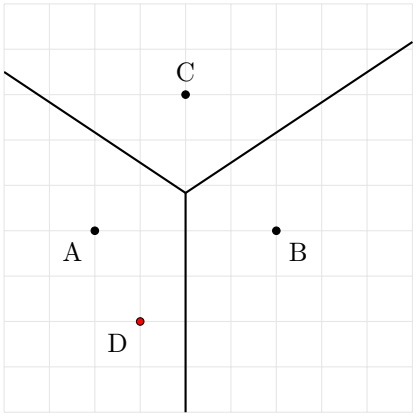
1. Find the equation of the perpendicular bisector of $[YZ]$.
2. Find the equation of the perpendicular bisector of $[XZ]$.
3. Find the coordinates of the Voronoi vertex V where the edges meet.

Ex 17: Consider three sites $A(3,2)$, $B(8,3)$, and $C(2,7)$.

1. Find the equation of the perpendicular bisector of $[AB]$.
2. Find the equation of the perpendicular bisector of $[AC]$.
3. Find the coordinates of the Voronoi vertex V where the edges meet.

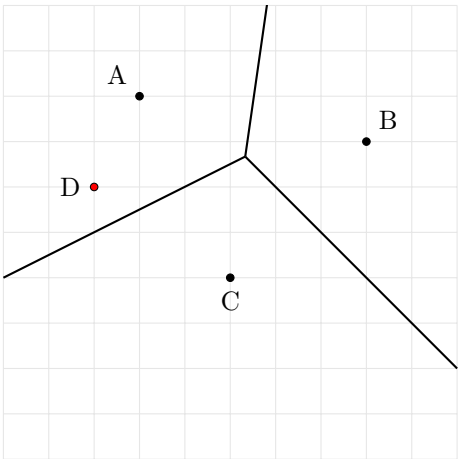
B.5 ADDING A NEW SITE

Ex 18: Site D is to be added to the Voronoi diagram shown below for sites A, B, and C.



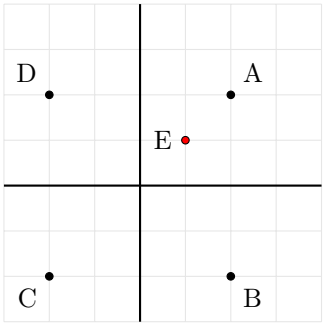
1. In which existing cell does site D lie?
2. Which of the existing cells will be affected by the introduction of site D?
3. Redraw the Voronoi diagram with site D added.

Ex 19: Site D is to be added to the Voronoi diagram shown below for sites A, B, and C. Note that triangle ABC is scalene (all sides are of different lengths).



1. In which existing cell does site D lie?
2. Which of the existing cells will be affected by the introduction of site D?
3. Redraw the Voronoi diagram with site D added.

Ex 20: Site E is to be added to the Voronoi diagram shown below for sites A, B, C, and D.



1. In which existing cell does site E lie?
2. Explain why site E will share a boundary with all other sites (A, B, C, D).
3. Redraw the Voronoi diagram with site E added.

C NEAREST NEIGHBOR INTERPOLATION

C.1 INTERPOLATING USING NEAREST NEIGHBOR ALGORITHM

Ex 21: A city has weather stations at $A(1,1)$, $B(5,1)$, and $C(3,5)$.
The temperatures recorded are: $A : 20°\text{C}$, $B : 22°\text{C}$, $C : 18°\text{C}$.
Estimate the temperature at point $P(2,2)$ using nearest neighbor interpolation.

1. Calculate PA^2, PB^2, and PC^2.
2. Which region contains P?
3. What is the estimated temperature?

Ex 22: Agricultural scientists measure the weekly rainfall (in mm) at three stations $R(1,5)$, $S(5,5)$, and $T(3,1)$.
The recorded values are: $R : 12$ mm, $S : 8$ mm, $T : 15$ mm.
Estimate the rainfall at a farm located at $P(4,4)$ using nearest neighbor interpolation.

1. Calculate PR^2, PS^2, and PT^2.
2. Which region contains P?
3. What is the estimated rainfall?

Ex 23: A biologist measures the soil pH at three locations $D(-2,3)$, $E(2,4)$, and $F(0,-1)$.
The pH levels are: $D : 6.5$, $E : 7.2$, $F : 5.8$.
Estimate the pH level at location $P(1,2)$ using nearest neighbor interpolation.

1. Calculate PD^2, PE^2, and PF^2.
2. Which region contains P?
3. What is the estimated pH level?

D THE TOXIC DUMP PROBLEM

D.1 OPTIMIZING LOCATIONS

Ex 24: The Voronoi diagram for four towns A, B, C, and D has two internal vertices located at $V_1(2,3)$ and $V_2(4,1)$. The coordinates of the towns are $A(0,2)$, $B(3,1)$, $C(4,2)$, and $D(4,0)$.
We want to locate a toxic dump as far as possible from any town.

1. Calculate the distance from vertex V_1 to its nearest site $A(0,2)$.
2. Calculate the distance from vertex V_2 to its nearest site $D(4,0)$.
3. Which of these two vertices is the best location for the toxic dump? What is the radius of the empty circle there?

Ex 25: A city currently has four fire stations located at $A(1,5)$, $B(4,2)$, $C(4,-2)$, and $D(0,2)$.
The city council wants to build a **fifth fire station**. To maximize efficiency and coverage, the new station should be located at a point that is as far as possible from the existing stations (filling the largest gap in coverage).
The Voronoi diagram for the current stations has two internal vertices located at $V_1(2,3)$ and $V_2(2,0)$.

1. Calculate the distance from vertex V_1 to its nearest station A.
2. Calculate the distance from vertex V_2 to its nearest station D.
3. Determine the coordinates of the optimal location for the new fire station E. Justify your answer.